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### Parallel Structures of Computer-Assisted Signature Pedagogy: The Case of Integrated Spreadsheets

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## **Parallel Structures of Computer-Assisted Signature Pedagogy: The Case of Integrated Spreadsheets**

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*This article was motivated by the authors' work on a project with a group of 2nd-grade students in a computer lab of a rural school in upstate New York. From this project, one goal of which was to provide a capstone experience for a teacher candidate in teaching application-oriented mathematics with technology, the ideas about parallel structures of two universes—the teacher's and the students'—that echo Shulman's structures of signature pedagogy have been developed. In the article the authors introduce the notion of computer-assisted signature pedagogy and explore its features through the combined lens of teaching and learning using the concept of Type I/Type II technology integration.*

The signature pedagogy construct introduced by Shulman (2005a) and explored for a variety of disciplines in an edited monograph (Guring, Chick, & Ciccone, 2009), can be characterized by three distinct levels called surface structure, deep structure, and implicit structure. The goal of the authors in this article is to explore the construct in the specific context of computer-assisted instruction, illustrated by the joint use of spreadsheets and other technologies in teaching mathematics to young children. It should be noted that, in general, the current signature pedagogy of mathematics both at the pre-college (National Council of Teachers of Mathematics [NCTM], 2000) and tertiary (Committee on the Undergraduate Program in Mathematics, 2004; Conference Board of the Mathematical Sciences, 2001) levels provides

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students with the opportunity of “doing mathematics rather than hearing about mathematics” (Ernie, LeDocq, Serros, & Tong, 2009, p. 265). This focus on *doing* mathematics through problem solving is the distinctive signature of a professional mathematician, whether one uses technology or does not.

The idea of developing mathematical habits of mind in students was introduced in North American secondary education half a century ago, and the following two quotes are from the most prominent sources: “To know mathematics means to be able to do mathematics” (Alfors, 1962, p. 189), and “for efficient learning, an exploratory phase should precede the phase of verbalization and concept formation” (Pólya, 1963, p. 609). Interestingly, teaching students to do mathematics is inherently linked to the following three descriptors of signature pedagogy by Shulman (2005b)—uncertainty (a problem might be too difficult to solve without, or even with, assistance), engagement (doing *is* engagement), and formation (regardless of the outcome of this engagement, one develops a kind of professional disposition toward the discipline). Currently, at any grade level, problem solving and mathematical exploration can be greatly assisted through the use of computers, which bring elements of uncertainty, engagement, and formation to the mathematics classroom. Therefore, these ever-changing teaching tools, as Shulman (2005a) noted, “create an opportunity for reexamining the fundamental signatures we have so long taken for granted” (p. 59).

In this modern technological context, one can talk about computer-assisted signature pedagogy (CASP). In CASP, the surface structure belongs to the level where a computer delivers a versatile learning environment that is visually stimulating and interactive. Although interactivity entails the computer’s response to one’s action that “sets the stage for interpretation, reflection and further action” (Kaput & Thompson, 1994, p. 678), teaching at the surface structure level is limited to students merely having ‘a good time’ from their actions. For example, a teacher’s goal may be to have students use a drill-and-practice program designed for a mathematics and/or a literacy lesson, or, at the very best, to create and visualize different images (iconic or symbolic) afforded by a specific computer application but not necessarily to expect that someone would start making connections among the images. Using computers at the surface structure level is what Maddux (1984) has termed a Type I application of technology in education, seeing it quite different from a Type II application—“new and better ways of teaching” (p. 38). More recently, Maddux and Johnson (2005a) argued that “the boring and mundane uses to which computers were often being applied [at the very outset of their educational applications] had set the stage for a major backlash against bringing computers into schools” (p. 2). This suggests that CASP cannot be viewed as an educationally successful tool unless a teacher employs other structure levels of signature pedagogy.

Encouraging reflection and supporting analysis of the action by a student implies that one teaches at the deep structure level. This kind of teaching requires broad pedagogical knowledge of what a computer environment

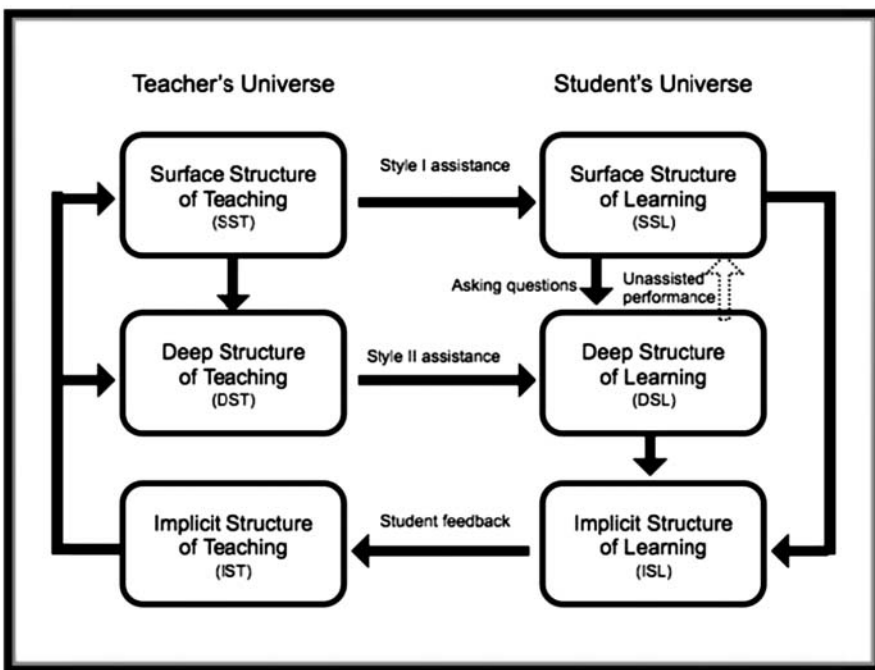
affords, intellectual courage to motivate students to reflect on their actions, readiness to answer unexpected questions, and willingness to support students' natural curiosity by helping them learn in the "zone of proximal development" (Vygotsky, 1987). This zone can be described as a dynamic characteristic of cognition that, in a problem-solving situation, measures the distance between two levels of the one's development as determined by independent and assisted performances. When a student's performance becomes assisted, the teaching occurs at the deep structure level. Teaching at that level also requires knowledge of national and state standards of the subject matter taught, understanding connections that exist between concepts that belong to different grade levels, and, more importantly, skills of using computers to support concept learning. When a computer is used at the deep structure level, Type II application of technology occurs (Maddux, 1984). The two types of educational computing should not be seen as "a simple dichotomy [between] bad and good practices" (Maddux & Johnson, 2005a, p. 3); rather, as suggested herein, the appropriate use of CASP may include a continuous move from one level to another influenced by a variety of instructional trends.

Teachers' beliefs about teaching, in general, and instructional uses of computers, in particular, are the major components of the implicit structure of CASP. An individual teaching philosophy either keeps the teacher at the surface structure level or motivates a quick transition from one level to another. Experience working at the deep structure level, in turn, affects the extent of the richness of the implicit structure of CASP. Similar to the two types of technology integration, one can talk about two styles of assistance that teachers can offer to their students. Style I assistance is typified by the surface structure of teaching, and it is limited by one's teaching philosophy which does not view teaching mainly as assisted performance (Tharp & Galloway, 1988). Style II assistance is typified by the deep structure of teaching, and it is open to promoting reflective inquiry and taking an intellectual risk by going into an uncharted territory brought to light through an open-ended classroom discourse. Likewise, a cursory knowledge of technology features by a teacher offers students Style I assistance only. By the same token, Style II assistance in the students' design and/or utilization of a computational learning environment requires a high level of technological literacy on the part of a teacher. These two styles of assistance, observed within the general instructional setting, underlie one's implicit structure of signature pedagogy and determine the composition of two other structures.

## PARALLEL STRUCTURES OF TEACHING AND LEARNING

The theoretical construct of signature pedagogy, besides being mapped to the domain of CASP, can also be extended to include the very recipients of the pedagogy, that is, students. This is consistent with the underlying

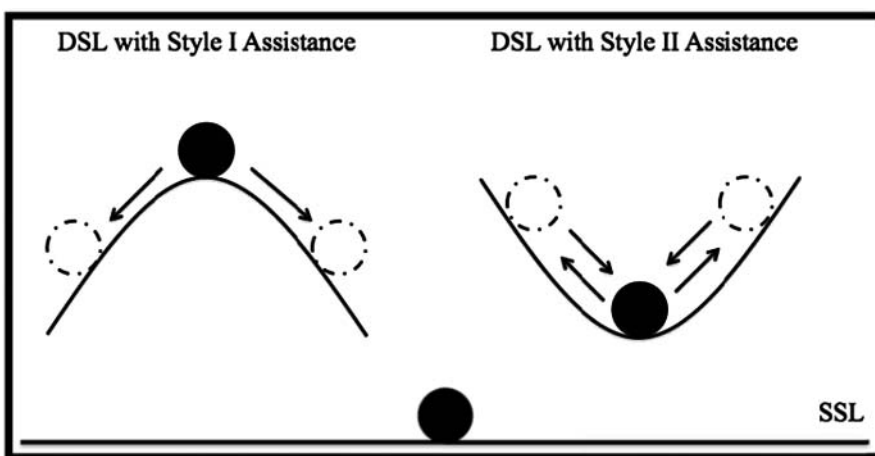
principles of educational scholarship, which sees the concept of signature pedagogy as a link between the theory of learning and the practice of teaching (Shulman, 2005a). In particular, in the context of mathematics education this concept is seen as a bridge between teaching and learning (Ernie et al., 2009). In general, the proposed extension, while having been motivated by the authors' work with a group of 2nd graders on a particular mathematics application project, is neither grade nor content specific and may be applied to any discipline the teaching of which can be computer assisted. Such a possibility, in turn, opens a window to future research and development in other contexts. As a result of bringing students into the picture of CASP, two separate but mutually dependent universes can be considered: teacher's universe and student's universe. Each universe includes three levels that echo Shulman's structures of signature pedagogy, which can be then considered as a part of the whole teaching and learning process. In this process, teaching affects learning and vice versa; that is, the way students learn (or want to learn) can alter the way teachers teach. Due to such reciprocity of teaching and learning, there exist the same three structures—surface, deep, and implicit—in the student's universe. As the concept map in Figure 1 shows, the teacher's universe includes surface structure of teaching (SST), deep structure of teaching (DST), and implicit structure of teaching (IST). By



**FIGURE 1** Parallel structures of teaching and learning: A concept map.

the same token, the student's universe includes surface structure of learning (SSL), deep structure of learning (DSL), and implicit structure of learning (ISL).

The teacher's and the student's universes not only contain parallel structures but constantly affect each other as both parties make their way through the levels of teaching and learning. Consider the case of CASP. The SST, perhaps inadvertently, keeps a student at the SSL level so that he or she would interact with a computer for the purpose of enjoyment only (Style I assistance which presupposes Type I technology integration only). However, the teacher cannot have full control of the student's use of and thinking about technology. For example, the latter might recognize patterns that the computer generates and then proceed to ask the former various questions about those patterns. In that way, the computer becomes a thinking device, thereby bringing the student to the DSL level (Type II application). However, the student's immersion into the DSL may be rather unstable, and the extent of its instability depends on the teacher's willingness, in turn, to enter the DST; in other words, it depends on what style of assistance a teacher is prepared to offer (Figure 2). If the student enters the DSL, but does not receive Style II assistance from the teacher, it is quite likely that he or she would exit it back to the SSL. Furthermore, receiving no support for intellectual curiosity affects one's cognitive disposition toward the continuation of having 'a good time' at the SSL level. This kind of a student's functioning within his or her universe is consistent with the dynamism of cognition expressed through the theoretical construct of the zone of proximal development. The longer both the teacher and the student function at the deep structure of their universes, in other words, the longer Style II assistance for Type II application of



**FIGURE 2** Unstable (left) and stable (right) equilibriums of CASP.

technology is provided, the more concept learning can result from the use of CASP.

By examining CASP through the combined lens of teaching and learning, one can recognize significant merits of the approach and its potential for achieving substantial learning outcomes. Consider CASP in the case of mathematics. A student's entrance into DSL may be motivated by a sudden recognition of a mathematical concept that a computer supports, be it by a teacher's design (manifesting DST) or not. In the student's universe, the ISL includes previous learning experiences and beliefs about what it means to learn and do mathematics (Ernie et al., 2009). Just like in the case of IST, the ISL affects both SSL and DSL. An example of this relationship is a student's belief that any mathematical model, be it symbolic or iconic, serves only a single problem rather than multiple problems. Even if the same model emerges in different contexts, this belief prevents one from recognizing the sameness, affects one's desire to move from SSL to DSL and thereby hinders conceptual understanding of mathematics. However, if a teacher functions at the DST level, he or she can guide a student to understanding that just like different problem-solving strategies can be applied to a single problem, different problems may be resolved through a single approach.

#### AN INTEGRATED SPREADSHEET AS AN ENVIRONMENT OF CHOICE FOR CASP

In order for CASP to become an effective didactic framework, one has to make right decisions regarding the choice of software to be used. For a project (described in more detail later) involving a group of 2nd graders, the authors—two education faculty members and a teacher candidate—have chosen an electronic spreadsheet. Designed originally for non-educational purposes, a spreadsheet was conceptualized by its inventor as a combination of an electronic blackboard and electronic chalk in a classroom (Power, 2000). Over the years, such pedagogical conceptualization proved to be an insightful prophecy as more and more successful classroom applications of the tool across grade levels and subject matters have been recorded. For example, it came as no surprise when the National Council of Teachers of Mathematics (NCTM, 2000) recommended using spreadsheets in the elementary classroom. Niess (2005a), one of the leading authors of the Technological Pedagogical Content Knowledge framework (Niess, 2005b; Niess et al., 2009) developed at the confluence of Shulman's (1986) notion of pedagogical content knowledge and Maddux's (1984) Type I/Type II concept, made a similar recommendation advocating the use of spreadsheets in the primary grades. These recommendations were originally based on and later motivated several studies dealing with the use of spreadsheets with young children as well as with their future teachers in various grade-appropriate mathematical

contexts (Abramovich, 2003; Abramovich & Cho, 2008, 2009; Abramovich, Stanton, & Baer, 2002; Ainley, 1995; Drier, 1999, 2001). In particular, in order to make the use of spreadsheets by young children more appealing, the research of the authors in this article extends earlier studies conducted at SUNY Potsdam by the first author (Abramovich & Cho, 2009) that showed promise as the application of CASP to the primary grades.

Toward this end, note that one can amplify the use of a spreadsheet as a teaching and learning tool by combining it with other modern technologies already familiar to younger users. These may include hand-held devices, such as the Nintendo DS, the PlayStation Portable, and the iPhone. In this way, while a student inputs and manipulates data through devices imbedded into a spreadsheet, the tool hides processing the data behind its amplified pedagogical image. In addition, one can create videos using Web-based type-to-text technology service Xtranormal (<http://www.xtranormal.com/>) so that students can get voice instructions from an entertaining video clip instead of reading them from the textboxes of a spreadsheet. From the point of view of CASP, the use of familiar images and tools of entertainment jointly with the computational and operational capability of a spreadsheet is important to lessen anxiety and timidity frequently felt by inexperienced computer users. It is this kind of environment (apparently not being described in the literature) that the authors refer to as an integrated spreadsheet and have used in their work with 2nd-grade students. As an aside, note that CASP, based on the use of integrated spreadsheets, can contribute to the improvement of invitational theory and practice (Purkey & Stanley, 1991).

#### SURFACE STRUCTURES OF TEACHING AND LEARNING WITH INTEGRATED SPREADSHEETS

The most obvious attribute of the SST in the case of an integrated spreadsheet is the visually engaging and informative nature of the learning environment which offers an attractive interface for young users in presenting information. Just like many word-processing and presentation software, a spreadsheet allows for a full array of fonts, shapes, and colors. This variety not only increases students' interest in technology but it provides an effective vehicle for information delivery. In addition to control of text font, size, and color, multiple types of shapes, outlines, colors, and backgrounds may be added. These shapes can be edited to include text boxes with problems to be solved. Additionally, spreadsheets allow pictures to be inserted into each worksheet as well as a background picture to be used. A picture is not only a means to increase student interest, reduce anxiety, and provide static information, but it also can serve as a virtual manipulative. Spreadsheet-based images can be used in all the same ways as physical manipulatives, but can also be labeled, deleted, copied, resized, and re-colored.



The surface structures of spreadsheets offer multiple ways of creating environments for teaching that can be tailored to individual learning styles, including those that address the need of students with disabilities. Although a teacher can design learning environments to encourage a variety of conventional reading and writing activities, spreadsheets can also be designed to emphasize nonverbal learning styles or multiple intelligences. Needless to say, such comprehensive uses of the software in a classroom by a teacher exemplifies Style II assistance that students receive when learning to use, and in some cases to design, a spreadsheet-based environment.

The use of multiple worksheets, either independent or connected, allow for the creation of learning environments of different levels of technical sophistication. Students can progress through the worksheets of a spreadsheet like through the pages of a book. The spreadsheet could also be designed to be a non-progressive educational environment so that students first choose a worksheet that appeals to them the most and continue to the others in any order. Furthermore, the zoom feature allows for text and pictures to be enlarged or minimized. The practical application of this feature is to adjust a spreadsheet to have all content appear on computers of varying screen resolutions. The educational application of this feature is to allow students with visual impairments or personal preference to increase magnification of objects and information presented by a teacher.

The spreadsheets can also be combined with other technologies, including physical manipulative materials and videos. Informative and entertaining videos can be used to represent concrete elements of a mathematical problem, deliver instructions for student actions, or introduce content in a variety of formats to accommodate different learning styles. Likewise, concrete materials can be used to support spreadsheet-based activities or provide a kinesthetic element to the learning environment. Both conventional and technological components of a lesson may be integrated with the spreadsheet to enhance teaching and learning.

#### DEEP STRUCTURE OF TEACHING AND LEARNING WITH INTEGRATED SPREADSHEETS

Shulman (2005a) defines the deep structure of signature pedagogy as the best way “to impart a certain body of knowledge and know-how” (p. 55). The value of using an integrated spreadsheet in support of CASP is that it relieves the demands of procedural understanding in pursuit of conceptual understanding. This feature allows younger learners to comprehend rather advanced (for the elementary classroom) concepts, particularly in mathematics. That is why, in the deep structure of CASP, the intent of the teacher should be to achieve conceptual understanding of mathematics. The deep structure of CASP includes the intention of instruction. Simply put, DST is

what we want students to do in order to gain comprehension, how we guide them to this comprehension, and what content is the goal. The greatest pitfall in the implementation of CASP is that students will either not progress to the DSL level at all or an extent of instability of their presence at that level would be high enough so that without a teacher being able to provide Style II assistance, an immediate return to the SSL level is the only option. Such a return, as was mentioned, would not be cognitively unnoticed and may result in the student's loss of interest in using a computer.

In order for CASP to take full advantage of teaching and learning with integrated spreadsheets at the deep structure of the two universes, an empirical approach to the development of knowledge can be brought to bear. This approach was strongly emphasized by John Dewey—the most notable reformer of the modern era of American education—who argued that experience is educative only if it results in one's intellectual growth. To this end, Dewey (1938) promoted the reflective inquiry approach to the development of knowledge. As was mentioned, reflection on experience occurs at the deep structure of both universes. Through a teacher-guided reflection occurring at the deep structure level of both universes, new knowledge can be developed, as students are encouraged to inquire about the meaning of their experience (Style II assistance). In the modern classroom, reflection can be motivated by the appropriate use of an integrated spreadsheet. Whereas the level of guidance and the use of the tool are determined by the teacher, “the ideas should be born in students' minds and the teacher should act only as midwife” (Pólya, 1981, p. 104). The spreadsheet, when used both by teachers and students at the DST and DSL levels, respectively, allows for a user-friendly, scaffolding computational experiment through which one can recognize a solution to a problem at hand before he or she is able to produce steps leading to its formal demonstration (Wood, Bruner, & Ross, 1976).

In the case of mathematics, CASP is most effective when it is embedded in a real-world context and supported by a physical model. However, without sufficient teacher guidance (Style II assistance), students can easily disconnect from this context. Even “playing” with numbers as a spreadsheet-based activity can be a significant predecessor of recognizing patterns that the numbers form. Experience developed at the SSL level when analyzed and reflected upon turns into new knowledge at the DSL level.

#### IMPLICIT STRUCTURE OF TEACHING AND LEARNING WITH INTEGRATED SPREADSHEETS

Shulman (2005a) noted that implicit structure of signature pedagogy includes “beliefs about professional attitudes, values, and dispositions” (p. 55). Can a spreadsheet be used with young children? How much time does it take

to teach young children basic spreadsheet skills? Can a spreadsheet be integrated with other technologies to enhance the user-friendliness of the tool? Can a spreadsheet be used as a problem-solving and/or problem-posing tool at the elementary level? A teacher's professional attitudes and beliefs regarding using technology in a classroom strongly affect a student's ability to learn.

In the case of mathematics, "technology influences the mathematics that is taught and enhances students' learning" (NCTM, 2000, p. 24), and it allows teachers "to construct new kinds of learnable and connected mathematics" (Noss & Hoyles, 1996, p. 251) for their students. That is, if a teacher's belief about the usefulness of a spreadsheet as a teaching and learning tool reflects the above views, the use of CASP by the teacher (that can be extended to include other disciplines) would turn a typically unstable equilibrium of a student's performance at the DSL level into a stable one.

For example, a teacher's IST may support a belief that computer fluency, in general, and spreadsheet proficiency, in particular, is an important skill for all students, including those in the primary grades. Then, CASP of the teacher will be directed toward increasing computer fluency of students. In particular, the teacher will be focusing on students' learning of essential computer skills such as creating and saving files, navigating directories, managing different platforms, etc. However, if a teacher views a spreadsheet as a tool for complex mathematical explorations only, that has little appeal for the development of basic mathematical concepts, the instruction would focus on the interactive and visual aspects of the tool, something that does not require significant skills to be developed.

#### MATHEMATICS, YOUNG CHILDREN, AND INTEGRATED SPREADSHEETS PROJECT AS AN EXAMPLE OF CASP

Contemplating about the best practices of teacher education in an interview with Falk (2006), Shulman noted that in order for the field to develop "systematic ways of preparing professionals . . . [it should be determined] what kind of curriculum materials, what kind of artifacts, what kind of technology—in the broad sense of technology—will need to be embedded to support [the notion of the signature pedagogy for teachers]" (p. 76). The next three sections demonstrate how a capstone practical activity for a teacher candidate that includes grade-appropriate curriculum, artifacts, and technology can be developed using the framework of CASP. Toward this end, the authors worked with four 2nd-grade students recommended by their classroom teacher for an after-school activity. Five 50-minute sessions were conducted in a computer lab of an elementary school in rural upstate New York. The activities were aimed at different goals: to help the school with the appropriate use of technology by students, to provide a capstone

learning experience in CASP for a teacher candidate (the second author), and to explore the use of integrated spreadsheets in the context of mathematical problem solving and posing.

The students were not familiar with a spreadsheet; and the skills such as using sliders and virtual manipulatives, typing information in the text boxes shaped as portable video games, and saving their work were taught as the activities progressed. They worked individually and were assisted by the authors as appropriate. Throughout the project, the students demonstrated a superb on-task behavior that can be ascribed to the enjoyable context of the activities (surface structures of teaching and learning), their natural drive for curiosity, the design of CASP that included reflection (deep structures of teaching and learning), and the authors' beliefs about what it means to do mathematics (implicit structure of teaching).

At the onset of the project, the third author questioned the rationale, beyond achieving the learning objectives, for having students solve mathematics problems. This question was deeply rooted in her IST, which holds the belief that whatever a student is asked to do should have a purpose. The search for a purpose led to the creation of a backstory for the project and videos of a robot guiding students from task to task. The integration of the videos in the traditional spreadsheet environment became extremely popular with the students and helped them to move from SSL to DSL in their learning universes.

One of the greatest facilities of a spreadsheet is that it allows students to explore the results of calculations and draw conclusions about mathematical concepts. This is particularly significant for young children who, thereby, can learn mathematical concepts prior to the development of formal procedural skills. For example, the concept of average requires one's proficiency in addition and division. The latter operation is not introduced until the 3rd grade at the earliest. Yet, an integrated spreadsheet was designed to allow students to increase or decrease the cell values by using scroll bars attached to cells and interactively display the average approximated to a whole number—the only number system that conforms to the 2nd-grade mathematics curriculum.

## PROBLEM POSING WITH INTEGRATED SPREADSHEETS

The National Council of Teachers of Mathematics (NCTM, 1989) has referred to problem posing as “an activity that is at the heart of doing mathematics” (p. 138) and noted “computer programs can engage students in posing and solving problems” (p. 76). Whereas engaging students in problem posing by using a computer is a clear-cut example of Type II application of technology, CASP requires a teacher to guide students in their move from SSL to DSL when posing a problem. Such guidance implies the need for Style II

assistance because in order to understand at which structure level of learning one uses technology for problem posing, this intellectual activity should be evaluated by analyzing student-formulated problems. In particular, Style II assistance requires a teacher's appreciation of the notion of didactic coherence of a problem (Abramovich & Cho, 2008) that includes numerical and contextual coherences as essential elements. To clarify, note that the authors observed how one of the students, excited by the possibility of using a slider to alter temperatures from zero to 100 degrees (SSL), formulated a problem of finding a five-day temperature range with the (computer-generated) answer of 95 degrees. Obviously, such a problem is not contextually coherent unless it was formulated in the context of a different planet from which the robot read his instructions. This example demonstrates how CASP not only provides students with a tool to pose a problem, but it assumes a teacher's readiness to provide them with Style II assistance in order to move from SSL to DSL in a problem-posing situation.

#### CONCRETE THINKING AS A BARRIER FOR UNDERSTANDING THE MULTIPLICITY OF ANSWERS

The importance of assisting students in complex and ambiguous situations, something that requires deep content knowledge, is not mathematics specific—the issue has been emphasized across the whole spectrum of signature pedagogies (Ciccone, 2009; Shulman, 2005b). In a computer environment, which is uniquely interactive, teacher's assistance implies the need to individualize teaching and learning if the potential of technology is to be realized (Maddux & Johnson, 2005b). This focus on individualization (requiring Style II assistance) was observed during the students' struggle with a new concept: the multiplicity of answers afforded by a single question. In a video created as an element of the integrated spreadsheet, a robot asked the students: *What could have happened with the temperature during a five-day period if the average temperature during this period was increased by one degree? Is there more than one answer? Why or why not?* The students responded that there was perhaps just one day when the temperature was increased by five degrees and, therefore, there is only one correct answer to this question. The authors' comprehension of what stood in their way to the conceptualization of the multiplicity of answers was critical. Without Style II assistance in answering the above questions the 2nd graders were unable to go beyond the concreteness of a single day and appreciate its possible variation over the five-day period, let alone the split of such an increase over several days. After being shown (using sliders) a number of ways of how the day (they saw as the single correct answer) can be any day, or how the one degree increase of average temperature can be split at least between two days, the notion of the multiplicity of answers was immediately

appreciated by the students as it turned out to be within their zone of proximal development. Still, the students were uncertain of how to manage the unfolding multitude of answers and needed assistance because, as Shulman (2005a) noted, “uncertainty produces both excitement and anxiety” (p. 57). Thereby, without ‘increasing the volume’ of Style II assistance, a seemingly stable equilibrium of CASP was in danger of bifurcating into an unstable one (Figure 2). With this in mind, while concrete *thinking* was found to be a barrier in the way of grasping abstraction by the students, concrete *activity* became the next step toward keeping them in the zone, or, alternatively, at the DSL level.

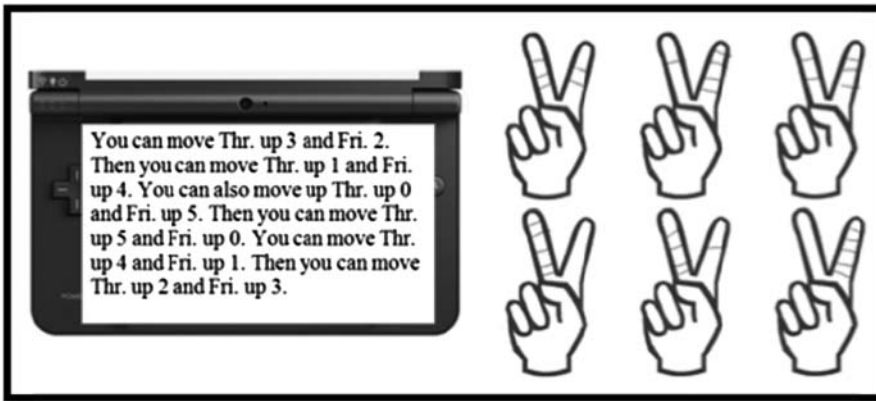
### CONCRETE ACTIVITY AS A MEANS OF FINDING MULTIPLE ANSWERS

In order to help students to experientially internalize the idea of multiple answers, the above set of questions was simplified both numerically and contextually to enable their full resolution through a hands-on activity: *How many ways can one split five pennies between two circles, or, alternatively, put five rings on two fingers?* The students were given coins and rings and were asked to record their experiential findings in the form of drawings and corresponding charts. Without much difficulty, they found six ways of carrying out both coins–circles and rings–fingers activities. The students not only found the equivalence between the two activities but used their drawings and charts to answer the question (presented by a robot and written in a textbox of a spreadsheet): *If the average temperature for the week increased by one degree, and the temperature on Monday, Tuesday, and Wednesday did not change, what temperature changes could have occurred on Thursday and Friday?*

In that way, the students were able to connect different contexts and effectively use their hands-on experience in a more abstract problem-solving situation. Figure 3 shows two isomorphic representations of answers to the last question by a student—through drawings (right) and through an integrated spreadsheet textbox (left). This individual aspect of teaching and learning with integrated spreadsheets, being an example of how CASP allows for the integration of Type II application of technology and Style II assistance, was conducive for the authors’ effective use of the DST components of the teacher’s universe in order to motivate the students’ work at the DSL level of their universe.

### CONCLUDING REMARKS

To conclude this article, note that one of the goals of the project from which the ideas of parallel structures of CASP and the associated concept of



**FIGURE 3** Isomorphism between arrangement of rings and weather changes.

Style I/Style II assistance have been developed was to provide a capstone experience for a teacher candidate (one of the authors) in teaching mathematics with technology. Because the project was contrived to be an extension of a pre-student teaching practicum focusing mainly on the observation of a traditional classroom, this experience included learning how to move from the apprenticeship of observation to the apprenticeship of teaching (Shulman, 2005a), more specifically, teaching with integrated spreadsheets. The latter type of apprenticeship was designed to offer experience in CASP with an emphasis on:

- how one can “make judgment under uncertainty ... [and] manage [students] levels of anxiety so that teaching produces ... learning” (Shulman, 2005a, p. 57);
- how one’s “understanding of what students know and need to learn” (NCTM 2000, p. 11) serves as a critical component of Style II assistance;
- how mathematics can be approached “from an experientially based direction, rather than an abstract/deductive one” (Conference Board of the Mathematical Sciences, 2001, p. 96);
- how to help students “learn mathematics with understanding, actively building knowledge from experience” (NCTM, 2000, p. 11);
- how spreadsheets can aid in “having children formulate problems themselves” (NCTM, 1989, p. 24); and
- how transforming “understanding of a topic into a graphical representation fosters the development of critical thinking skills” (MacKinnon & Keppell, 2005, p. 312).

The use of CASP in the context of integrated spreadsheets and weather changes, required from the teacher candidate making an informed decision

of when to answer and when to avoid answering questions asked by the students. For example, after it was found that five rings can be put on two fingers in six ways, the students were asked to find as many ways as they possibly could to put those rings on three fingers. “Tell us the magic number!” shouted the students out of curiosity. When given the answer, 21, they were excited—“I won!” exclaimed one of the students who, by reaching already the needed level of abstraction, managed to find eight ways to represent five as a sum of three whole numbers; in other words, to put five rings on three fingers. This, in turn, resulted in a self-posed homework to actually experience the 21 ways. At the same time, the students were not told that there are 126 ways to raise average temperature by one degree over five days as this number is far beyond one’s ability to be verified through a hands-on activity.

The parallel structures of CASP in which two universes—the teacher’s and the student’s—were shown to be controlled by two types of assistance and two types of technology integration. Forming a kind of a concept map (Novak, 1998), such a conceptualization of teaching and learning with computers can be extended beyond the primary level mathematics and applied to any discipline and grade level. Regardless of what is being taught and who is the learner, using computers has the potential to keep one’s interest in learning at the deep structure of his/her universe. As the authors have demonstrated in this article, successful computer integration requires many efforts both from teachers and students, and the authors’ ideas about CASP were presented as an example of how those efforts can be coordinated in a didactically meaningful way.

## REFERENCES

- Abramovich, S. (2003). Cognitive heterogeneity in computer-mediated mathematical action as a vehicle for concept development. *Journal of Computers in Mathematics and Science Teaching*, 22(1), 19–41.
- Abramovich, S., & Cho, E. (2009). Mathematics, computers, and young children as a research-oriented learning environment for a teacher candidate. *Asia Pacific Education Review*, 10(2), 247–259.
- Abramovich, S., & Cho, E. (2008). On mathematical problem posing by elementary pre-teachers: The case of spreadsheets. *Spreadsheets in Education*, 3(1), 1–19.
- Abramovich, S., Stanton, M., & Baer, E. (2002). What are Billy’s chances? Computer spreadsheet as a learning tool for younger children and their teachers alike. *Journal of Computers in Mathematics and Science Teaching*, 21(2), 127–145.
- Ainley, J. (1995). Re-viewing graphing: Traditional and intuitive approaches. *For the Learning of Mathematics*, 15(2), 10–16.
- Alfors, L. V. (1962). On the mathematics curriculum of the high school [Memorandum]. *American Mathematical Monthly*, 69(3), 189–193.
- Committee on the Undergraduate Program in Mathematics. (2004). *Undergraduate programs and courses in the mathematical sciences: CUPM curriculum guide*



2004. Washington, DC: The Mathematical Association of America.
- Conference Board of the Mathematical Sciences. (2001). *The mathematical education of teachers*. Washington, DC: The Mathematical Association of America.
- Ciccione, A. A. (2009). Foreword. In R. A. R. Guring, N. L. Chick, & A. A. Ciccione (Eds.), *Exploring signature pedagogies* (pp. xi–xvi). Sterling, VA: Stylus.
- Dewey, J. (1938). *Experience and education*. New York, NY: MacMillan.
- Drier, H. (1999). Do vampires exist? Using spreadsheets to investigate a common folktale. *Learning and Leading with Technology*, 27(1), 22–25.
- Drier, H. (2001). Teaching and learning mathematics with interactive spreadsheets. *School Science and Mathematics*, 101(4), 170–179.
- Ernie, K., LeDocq, R., Serros, S., & Tong, S. (2009). Challenging students' beliefs about mathematics. In R. A. R. Guring, N. L. Chick, & A. A. Ciccione (Eds.), *Exploring signature pedagogies* (pp. 260–279). Sterling, VA: Stylus.
- Falk, B. (2006). A conversation with Lee Shulman—signature pedagogies for teacher education: Defining our practices and rethinking our preparation. *New Educator*, 2(1), 73–82.
- Guring, R. A. R., Chick, N. L., & Ciccione, A. A. (Eds.). (2009). *Exploring signature pedagogies*. Sterling, VA: Stylus.
- Kaput, J. J., & Thompson, P. W. (1994). Technology in mathematics education research: The first 25 years in the *JRME*. *Journal for Research in Mathematics Education*, 25(6), 676–684.
- Maddux, C. D. (1984). Educational microcomputing: The need for research. *Computers in the Schools*, 1(1), 35–41.
- Maddux, C. D., & Johnson, D. L. (2005a). Type II applications of technology in education: New and better ways of teaching and learning. *Computers in the Schools*, 22(1/2), 1–5.
- Maddux, C. D., & Johnson, D. L. (2005b). Information technology, Type II classroom integration, and the limited infrastructure in schools. *Computers in the Schools*, 22(3/4), 1–5.
- MacKinnon, G. R., & Keppell, M. (2005). Concept mapping: A unique means of negotiating meaning in professional studies. *Journal of Educational Multimedia and Hypermedia*, 14(3), 291–315.
- National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Council of Teachers of Mathematics (NCTM). (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- Niess, M. L. (2005a). Scaffolding math learning with spreadsheets. *Learning & Leading with Technology*, 32(5), 24–25, 48.
- Niess, M. L. (2005b). Preparing teachers to teach science and mathematics with technology: Developing a technology pedagogical content knowledge. *Teaching and Teacher Education*, 21(5), 509–523.
- Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper, S. R., Johnston, C., . . . Kersaint, G. (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, 9(1), 4–24.
- Noss, R., & Hoyles, C. (1996). *Windows on mathematical meaning: Learning cultures and computers*. Dordrecht, The Netherlands: Kluwer Academic Publishers.
- Novak, J. D. (1998). *Learning, creating, and using knowledge: Concept maps as*

- facilitative tools in schools and corporations*. Mahwah, NJ: Lawrence Erlbaum.
- Pólya, G. (1981). *Mathematical discovery: On understanding, learning, and teaching problem solving* (combined ed.). New York, NY: Wiley.
- Pólya, G. (1963). On learning, teaching, and learning teaching. *The American Mathematical Monthly*, 70(6), 605–619.
- Power, D. J. (2000). DSSResources.COM. Retrieved from <http://dssresources.com/history/sshistory.html>
- Purkey, W. W., & Stanley, P. H. (1991). *Invitational teaching, learning, and living*. Washington, DC: National Educational Association Library.
- Shulman, L. S. (2005a). Signature pedagogies in the professions. *Daedalus*, 134(3), 52–59.
- Shulman, L. S. (2005b) *The signature pedagogies of the professions of law, medicine, engineering, and the clergy: Potential lessons for the education of teachers*. Paper presented at the National Science Foundation Mathematics and Science Partnerships Workshop: Teacher Education for Effective Teaching and Learning. Irvine, CA: National Research Council's Center for Education.
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(4), 4–14.
- Tharp, R. G., & Gallimore, R. (1988). *Rousing minds to life: Teaching, learning, and schooling in social context*. Cambridge, MA: Cambridge University Press.
- Vygotsky, L. S. (1987). Thinking and speech. In R. W. Rieber & A. S. Carton (Eds.), *The collected works of L. S. Vygotsky* (Vol. 1, pp. 39–285). New York, NY: Plenum Press.
- Wood, D., Bruner, J. S., & Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, 17(2), 89–100.