National Council of Teachers of Mathematics. (2000). *Principles and Standards for School Mathematics*, Chapter 4: Standards for Grades Pre-K-2. Reston, VA: Author.

Summary

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Chapter 4 of this publication focuses solely on mathematics for the youngest learners, specifically the standards for grades Pre-K – 2. This publication emphasizes the importance of mathematical development in children from birth to the age of four and how integral exposure to mathematical learning is for obtaining a cognitive foundation in mathematics. Introducing young children to mathematical concepts throughout their youngest years not only prepares them academically for school but allows them to experience challenges at a young age which subsequently develops their ability to explore ideas related to shapes, patterns, and numbers with growing sophistication. Much of a child's preparation for school will occur within the informal experiences. Informal does not imply a lack of sophistication or organization, but rather implies that mathematical growth can be achieved in everyday experiences and in environments in which thinking, creativity, exploration, are highly encouraged.

Play is children's work. It is often misconstrued that learning, specifically of mathematical topics occurs at definite times segregated from times of play. However, children learn through their everyday experiences and are more likely to retain information and develop a disposition for mathematics when it is integrated in activities and situations in which they are interested in. In this respect, almost every situation can become a vehicle for developing mathematical thinking and reasoning. For instance, something as simple as counting a child's crackers aloud, arranging stuffed animals by size, or cutting sandwiches into various geometric shapes are examples are everyday activities in which math can be accentuated. There are also various examples found outside the home, such as the shapes of buildings and signs, counting steps, recognizing patterns in daily routines, and following directions that reinforce this concept of drawing correlations and making connections from the real world to mathematics. Mathematical language should also be used frequently throughout informal

experiences, such as comparing the height of various children, or referring to the exact number of objects or the same number of objects in a given situation.

Most importantly, since these are impressionable, integral years in a child's life in regard to cognitive development, it is important that these early experiences are approached and framed in a positive light. The support of parents and guardians at this time cannot be understated. When children first enter Pre-K, their hypothetical lack of knowledge is often a reflection of the lack of opportunity they had prior to school, rather than their inability to learn.

Number and Operations

Pre-K – 2 places an influential emphasis on the concepts and skills related to number and operations. During this time, students are taught to enhance the sophistication of their counting skills using multidigit computation strategies. The teacher must focus on refining students' sense of numbers by moving from the initial basic counting to comprehension of the size of numbers, relationships within numbers, patterns, operations, as well as place value. A goal within this timeframe will be for students to develop flexibility within numbers, such as representing a quantity in various ways. Developing an understanding of the base-ten numeration system and place-value concepts are expected by the end of Grade 2. It is also important for teachers to approach these lessons using interesting tasks or examples, which will actively engage students in mathematical thinking and reasoning.

Algebra

Algebra is a subject that will continue to develop and evolve during Pre-K to grade 2 and, in fact, had already started developing at an earlier age. Repetitive rhymes, interactive songs, and rhythmic chants have already begun developing recognition and analysis of patterns. Students will begin to notice that operations are specific properties which causes them to make algebraic correlations. For

example, students will notice that changing the order in which two numbers are added will not affect the product. Students will also develop an understanding of being flexible when it pertains to patterns. For instance, patterns can be assigned official forms, such as AB or AABA, to represent pattern formations they observe. The use of skip-counting, hundreds charts, and charts for organizing information are useful in displaying and organizing questions and algebraic symbols in various ways. The use of manipulatives when making models is also an exemplar method to represent and understand quantitative relationships.

Geometry

Students already come to school with an informal understanding of geometric shapes and spatial knowledge. It is the responsibility of the teacher to expand that previous knowledge through exploration and investigations of shapes and structures in class. Not only should students widen their grasp of these concepts, but students should develop their own vocabulary to describe objects and differentiate them. Geometry will also lend well to developing relationships to concepts such as distance, coordinate geometry, navigations and maps. Classroom activities should also enhance the students' visualization techniques, such as requesting students to imagine something in their minds eye before subsequently drawing it or modeling it in some other way. Spatial visualization forge links among geometry and measurement through abstract techniques that solidify the students' understanding of tangible objects, such as having an understanding of how many skittles are in a jar based on one's understanding of how much space one skittle takes up on its own.

Measurement

Measurement forges both geometry and numbers, while simultaneously teaching everyday skills and measurement concepts that students will use for the remainder of their academic career and lives. Before proceeding to complex

forms of accurate measurement, students should begin via the concept of general comparison. For instance, deciding which shoe is larger than the other, or which child is taller than the other. This conceptual understanding of size, length, and width must be formulated in the early years. Teachers must provide ample opportunities for students to explore and take advantage of hands-on opportunities using various units of measurement. These units may be standard or non-standard, such as using pencils to measure the length of the doorway. An early application of number sense would also include estimation activities.

Data Analysis & Probability

While the types of activities appropriate for Pre-K to grade 2 students will greatly vary, informal activities that involve comparing, classifying, and counting will provide students the foundation necessary for understanding data, data analysis, and statistics. The formulation of student questions will serve as the basis for collecting data. This natural inclination to pose questions will simultaneously serve as a form of student engagement and interest, especially when these questions revolve around topics of interest. A child's curiosity will subsequently serve as the pivot-point from posing questions to collecting data, analyzing data, and making decisions as well as future predictions. Students should learn that the method of collecting data will differ depending on the questions posed as the process of refining questions, considering alternative ways of displaying information, or collecting information will not come to students automatically, but through practice, experimentation, and experience. By the end of Grade 2, students should have an understanding of misconceptions and extraneous variables when collecting and displaying data. For instance, a student may be executing a survey to see who watches the same television show, but if she only asks her close friends, she is not collecting an accurate depiction of who in the class watches the show in question. Teachers should also begin the discussions of probability through the informal use of dice, spinners, and other games of chance.

Problem Solving

Problem solving builds on a child's innate inclination to find a solution to a problem or posed question. It is the teacher's responsibility to pose a multitude of various questions while helping students to identify the essential information, so students may organize their thoughts. For instance, a teacher may choose to have the class plant a garden in the school yard. However, every student must have the exact same amount of space within the specified measurements of their garden. Students must first address their pressing questions before moving on to drawing a map, taking into account the natural barriers of the school yard, as well as creating a model that accurately depicts the measurements of their respective space. The problem solving required to share land as a class is an example of a classroom-based problem-solving strategy. There are several countless opportunities for various subject-matter integration when supplementing mathematical problem-solving. Integrating children's literature, such as the reading of a novel about animals, and calculating the total number of animals is an example, for instance. Teachers should ask students to reflect, explain, and justify their responses with clear evidence. Not only does this affirm the teacher's assessment of understanding, but it confirms the students' understanding.

Reasoning & Proof

Students' reasoning develops before school begins and is perpetually refined and modified by their new experiences. These new experiences in the classroom help students reason systematically, especially when encouraged to make conjectures, and are given ample time to search for evidence that proves or disproves their ideas. Formal reasoning is an important skill that can be developed through stating their reasoning. For instance, a student claiming to their teacher

that they have created two patterns in one by modelling two squares, two triangles, and two circles, while also alternating between a large shape and a small shape. Reasoning about classification will evolve drastically from Pre-K to Grade 2. For instance, a kindergarten student may claim that a triangle is considered large and categorize it as large, and another student may analyze the three sides and categorize it as a triangle. Once students have reached Grade 2, it should be clear that shapes have multiple properties and forms of classification will include multiple properties as well. It will be the teacher's responsibility to encourage verbal reasoning and justification, while also correcting and demonstrating mathematical language. Their justification may be correct, but the language could be incorrect or ineffective. The teacher may rephrase a question to determine exactly what a student was referring to, which will lead to the refining of the answer submitted. The use of manipulatives cannot be understated at this time. Students must know that they are capable of understanding any mathematical concept and should use their learning environment to their advantage in order to thrive.

Communication

Language is a powerful tool and the vehicle for which mathematics is communicated and fostered. Students, like adults, communicate in a variety of ways using verbal communication, hand gestures, pictures, objects, tools, and non-verbal communication. In order to accurately observe a student's mathematical process, the teacher must be able to observe the student via a form of communication, which then accommodates further refining and development of their reasoning. An important step in communication is for students to learn to clarify their ideas. Students must transition from wanting *more* milk to wanting another *glass* of milk. Students must transition from stating the triangle is *bigger* to stating the triangle is *twice* as large. A helpful strategy is to teach students to listen intently to each other and each other's ideas, as well as question each other in order to make learning advances. It is the teacher's responsibility to challenge students, encourage their problem-solving skills, while requesting that they continuously explain and justify their reasoning and responses. A multitude of

opportunities should be offered to students to think about a concept and discuss it with their peers. The goal should be to create a community of learners in which every student feels comfortable learning and expresses mathematical ideas, not only with the teacher, but with each other.

Connections

As discussed previously in this publication, students had already begun their mathematical learning informally in their early years before school commenced. An important connection for young learners is the connection between the informal learning obtained from home and the mathematics they learn in school. Understanding this integral connection erases the invisible barriers that separate mathematics learned in school and mathematics learned elsewhere. The child's mathematical abilities increase significantly when they understand that all learning is valuable and correlations may be drawn regularly from life experience in the classroom. When a kindergarten student responds to your question of how old they are with five fingers, they are demonstrating an understanding of the connection between their age and the number of fingers they are presenting. It is the responsibility of the teacher to further facilitate these connections by providing the necessary information to bridge the gap. It is also the teacher's responsibility to inform students that mathematics can be found in everyday activities and students should seek to find them throughout the da. For instance, students may be asked to count the number of times they can jump rope successfully or how fast they can run from one end of the gym to the other. Seeing the usefulness of using mathematics in everyday activities will enhance the students' success in other situations requiring mathematical solutions. In general, students should understand that learning is fluid and they should be open-minded to all mathematical learning opportunities they come across.

Representation

Representations are methods of communication that develop as students' progress in their years of school. Students use representations to organize their thinking and allow students to record their steps in a process. For instance, when answering a question as to how many wheels are on four bicycles and three tricycles, a student may draw four bicycles and three tricycle wheels. The student then no longer carries the burden of having to count and retain the count in their head for they have recorded it on paper and can then count out the wheels before revealing the product. A responsibility of the teacher is to create a learning environment in which one concept may be represented in a multitude of ways. Students should be encouraged to share various representations in their responses as well as consider alternative ways of thinking and different perspectives. Teachers are responsible for alerting students to the fact that representations can be interpreted in various ways and that representations best supplement a communicated response.