The Mathematical Education of Teachers: Traditions, Research, Current Context

This article begins by outlining the relevance of traditional beliefs and practices within mathematics departments. In the 1980s, researchers began to document the observations of students K-12 and the types of “math minds” that exist within our classrooms. These observations circled around types of individuals who were “strong” in math or those simply “ordinary.” According to this research, the difference between these two types of students lies solely around the level of understanding when it comes to mathematics. Those who are able to solve a math problem in five minutes or less would likely have an extensive comprehension while those students who simply apply memorized facts in order to solve the same problems may not have the existence of the “math mind.” As a result of this research perspective, teaching styles and approaches have mediated mathematics in the classroom.

With this being said, there has been a large influence on traditional mathematics teaching practices that have shaped perspectives of both students and teachers today. From the colonial period to the 1820s, the “rule method” was one approach that very much dominated the mode of instruction. This method followed the ideology that one would “memorize a rule, then practice using it” (p. 9). This was further built upon in the 1920s when memorization was at large within the mathematics classroom. In memorizing the rules, one would ideally have success in finding the answers, although, this did not lead to any aspect of finding “relationships” between various ideas. These pedagogical approaches did not seem to serve students well, and especially not
teachers. At this time, there were little to no additional mathematical requirements for K-8 teacher certification, with the view that they learned all that they needed to during their own elementary schooling, however, this was found to be limiting. In recent years, prospective teachers have been challenged to not only learn the relevant mathematical procedures to teach their students, but have been encouraged to practice mathematics in ways of actually “doing” mathematics.

Furthermore, teacher effectiveness research has often been analyzed through two aspects: teacher knowledge and student achievement (e.g. standardized tests). Teacher knowledge, however, can then be examined in several different ways including certification status as well as number and type of mathematics courses taken, however, it can be noted that these areas of measurement can potentially be misleading. This concept led to the research of “knowledge for teaching,” beginning in the 1980s, focusing more on the action teachers were taking in the classroom and the use of manipulatives, for example, with less focus on the test scores of the students. As one might conclude, the aforementioned approach to mathematics was a better predictor than one’s certification status or classroom test scores.

An additional teaching approach that had been studied involved that of learning trajectories. For example, taking an aspect of the curriculum and breaking it down into smaller assessments through step by step tasks contributed to a significant increase in student understanding. In all, focusing on the stepping stones required to achieve a goal is a much better predictor of student success rather than simply requiring students to “memorize.” Learning paths and effective teacher knowledge ultimately come together to improve mathematics learning and understanding – one cannot occur without the other to be successful. Mathematicians have an integral role in the success of teacher education, curriculum and assessment and it would be difficult to develop relevant structure of mathematics without their expertise.
Recommendations: Mathematics for Teachers; Roles for Mathematicians

In addition to the traditional beliefs and practices within the educational system, the mathematics for teachers and the role of mathematicians are two other categories that display a significant importance within this article. First beginning with mathematics for teachers, there are several recommendations that stand out for teachers that instruct mathematics in any way, shape or form. It is first recommended that teachers take mathematics courses that will give them the understanding of the mathematics they will be teaching, which closes ties in with the second outlining a suggested number of hours a teacher should spend depending on the grade and academic level. For example, a high school mathematics teacher is recommended to have many more hours of mathematics and at an undergraduate major level as opposed to a third grade general education teacher who will need less hours due to a focus of more fundamental concepts.

An additional recommendation outlines the importance of teachers in taking part in continued professional growth in mathematics over the course of their careers. Mathematicians have an important role within this aspect as they are the drivers towards assisting teachers in transitioning into different grade and experience levels. This would include anything from short courses on mathematics, to professional experiences and teacher-mathematician partnerships – all contributing to the overarching success of teaching mathematics.

The final recommendation for teachers in particular, encompasses many of the aspects from the previous three recommendations. Recommendation four states the necessity that all courses and professional development should attribute to a teacher that is developing a mind of problem-solving and that developing this habit that can then be transmitted to the students. It is important that no matter the experience level of the teacher, in order to actually teach, one needs to experience the learned concepts within their own education.
There are an additional two recommendations (#5, #6) outlined in the article although these fall under the category of the role for mathematicians. Recommendation five, states the crucial reality for teacher education courses in mathematics to be taught in collaboration with mathematics education faculty. For instance, such courses must be led by a faculty member that can instruct effectively for both the purpose of teacher education and mathematics. Overall, the content and teaching strategies of mathematics are at the core of importance for future and present teachers, and with that, experienced faculty for teacher education should be a priority.

Finally, the sixth recommendation under the role of mathematicians describes the need for partnerships between faculties of mathematics and faculties in mathematics education. This concept has recognizable improvements in terms of teachers themselves, as well as state, regional and school-district leaders. For mathematics to strive to reach its extent of effectiveness, all parties involved in education need to be on the same page and at the same level of understanding.

The final section of this article contains the focus of elementary teachers and the in depth analyzation of how teacher preparation involves a study of the math that will be taught, from the perspective of a teacher. Teaching mathematics is much more than relying on one’s past experiences to be successful. There is a demand for future teachers to understand the comprehensive approach to this subject across all grade levels, for example, connecting concepts from first grade math to that of sixth grade. It is important to note that the Common Core State Standards not only outline the mathematical standards that must be practiced for grades K through 12, but they also highlight the written application for elementary teachers. With this concept in mind, there are some essential ideas for teachers grades K-5 (one of the cohorts of focus within
the article) and the knowledge of how these ideas connect to mathematics in the later grades is of equal importance to be aware of.

To analyze the aforementioned concept more in depth, the example from the article will be used. When looking at “Counting and Cardinality” for kindergarten (pg. 25), there is the concept of learning how to count as a list of numbers in a sequential order. Within this concept, the illustrative activity states “examine counting errors that young children typically make and study the learning path of counting,” and following this are the standards. The next concept is for kindergarten to grade five called “Operations and Algebraic Thinking” (pg. 26). This concept involves different types of problems such as addition, subtraction, multiplication and division as well as the differentiation of these problems based on the operation. It can then be noted that there is a connection between these two learned concepts. In order to use an operation, one must understand the path of counting in order to properly solve a problem involving an operation. There are other concepts including “Number and Operations in Base Ten,” “Numbers and Operations – Fractions,” “Measurement and Data,” and “Geometry” all for kindergarten to grade five, building off one concept to further learn another.

Additionally, since there has been much focus on prospective teachers, it cannot be overlooked that those who have been practicing teaching for an extended period of time also benefit from additional studies of mathematics. For prospective teachers, studies should focus on 12 semester hours from the perspective of the teacher and involve the connections that all fundamentals have on one another, as described in the previous paragraph. For practicing teachers, opportunities to strengthen their knowledge of the content itself, and the minds of their students is integral to their success. This can be completed in a variety of ways including teacher groups of the same grade level focusing on a specific domain of mathematics, teacher groups across grade
levels looking at various domains and how they all connect or a group of teachers may be taught by a mathematics specialist to understand how to refine their lesson plan practices. Ultimately, these different approaches to further one’s mathematical teaching knowledge should focus on the delivery of content and the minds of the students, regardless on the mode being practiced.

To tie everything together, this development of teaching expertise should be completed for the opportunity of the teacher to be able to work through problems along with the enjoyment and satisfaction of understanding the subject of mathematics. None of this can become a reality without a perspective of open-mindedness to learn new fundamental approaches. In the Common Core State Standards for mathematics, there often becomes several solutions to one problem and, therefore, the examination of new ideas becomes a crucial phenomenon for practicing teachers. For example, technology is a powerful tool when it comes to teaching mathematical concepts in the classroom. Professional development for experienced teachers then needs to provide opportunities in order to put these innovative strategies to practice in order for all students to benefit from a deeper understanding of mathematics. Expanding one’s realm of expertise, in technological approaches for example, allows the teacher to provide several measures of problem solving techniques as well as a supplemental mode of learning. Overall, it is important for both prospective and practicing teachers to utilize mathematics specialists within the school if they are present. These individuals act as supports, resources and mentors that allow for improvement and wholesome learning to occur.