Teaching and Learning Mathematics

What do we know and what are the implications for the school mathematics classroom?

Good teaching
is more than just
telling or explaining.

Effective learning
is more than just following
a series of steps.

The Teaching Principle

Effective teaching requires understanding what students know and need to learn and *challenging* and *supporting* them to learn it well.
(NCTM, *PSSM*, 2000)

The Learning Principle

Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.

(NCTM, *PSSM*, 2000)

Effective learning is about *understanding*.

Two Kinds of Understanding:

(Skemp, 1967)

Instrumental:

- Having and using rules, but without reasons
- Advantages: (?)
  - Easier to understand
  - More immediate rewards
  - Quicker answers

Relational:

- Knowing both what to do and why
- Advantages:
  - Adaptable to new tasks
  - Easier to remember
  - Effective goal in itself
  - Schemas are organic

What does it mean to **understand**?

What does it mean to understand?

- Discuss and ask questions
- Apply to contexts other than the original
- Make connections and see relationships
- Access knowledge readily / understand question asked
- Understand how a concept works and what it means

"The portrait of human learning that is emerging from the new sciences of learning suggests approaches to pedagogy, instruction, curriculum, and assessment that differ significantly from those common in today's schools." (NRC, 2000)

Learning Sciences:

- Cognitive psychology
- Developmental psychology
- Learning and transfer studies
- Social psychology
- Brain/neurosciences research
- Technology-based learning environments

One of the most important influences on contemporary learning theory comes from basic research on how experts learn and think in contrast to the way novice learners approach new tasks and go about solving problems.

Experiments with Chess Players (deGroot, 1965)

- Studied expert (master) vs. inexperienced vs. novice chess players
- Compared accurate recall for game pieces in a meaningful game configuration vs. random patterns of pieces
- Expertise led to much higher recall when board configuration was meaningful, but not when random

Experts

- Notice and remember features and important, meaningful patterns
- Organize content knowledge around big ideas in context, not isolated facts
- Flexibly retrieve important aspects of their knowledge with little attentional effort
- Are flexible in approaches to new situations
- Can think about what they know and do not know (NRC, 2000)

Novices

- Do not notice patterns or features of patterns
- Have no systematic way (framework) to make sense of information
- Search for correct formulas and pat answers
- Cannot retrieve important information easily and spend time re-learning instead of remembering or reconstructing
- Do not recognize what they do not know (NRC, 2000)

Cognitive Activity and Structure of Knowledge

Cognitive Activity	*Fragmented Structure*	*Meaningful Structure*
Problem Representation	Surface feature Shallow	Underlying principles
Strategy Use	Trial and error	Efficient and goal-oriented
Self-monitoring	Minimal	Ongoing
Explanation	Simple facts	Coherent concepts

To develop competence, students must:

- Have a deep foundation of factual knowledge
- Understand facts and ideas in context of a conceptual framework
- Organize knowledge in ways that facilitate retrieval and application (NRC, 2000)

Are you a surface learner or a deep learner?

Surface learners:

- Attempt to "cram" facts into their heads
- Focus on short-term memorization of facts, formulas and concepts
- Assume this is what they will be tested on

Deep learners:

- Seek meaning when studying
- Reflect on what they read and hear
- Undertake to create their personal understanding of material

Early Cognitive Development

- Young children actually make sense of their worlds. In some domains (e.g. language, number) they seem predisposed to learn.
- Children are "ignorant," but not stupid.
- Children are natural problem-solvers and seek out novel challenges.
- Children develop knowledge of their own learning capacities very early.
- Children's natural capabilities need assistance (catalyst and mediation) for effective learning.

Learning and Transfer

- Hope: Students will transfer learning from one problem to another, one course to another, one year to another, and from school to beyond (home, work, etc.)
- All new learning involves transfer based on previous learning/ experiences.

Research on Transfer

- People must achieve a threshold of initial learning that is sufficient to support transfer.
- Spending a lot of "time on task" is not in and of itself sufficient to ensure learning.
- Learning with understanding is more likely to promote transfer than memorizing information from a book or lecture.

Learning and Transfer

- Knowledge that is taught in a variety of contexts is more likely to support flexible transfer.
- Knowledge that is overly contextualized can reduce transfer.
- Sometimes the knowledge that people bring to a new situation impedes learning because it guides thinking in wrong directions.

Research on Transfer

- Students develop flexible understanding of when, where, why and how to use their knowledge to solve new problems if they learn to extract underlying principles from their learning exercises.
- **The bottom line**: Transfer is an active, dynamic process. (NRC, 2000)

Insights from Brain/ Neuroscience Research

- Learning changes the physical structure of the brain.
- There are some critical periods for learning (e.g. phonemic perception, language learning).
- Brain development is a function of biologically-driven processes in combination with lived experience.

Brain Research Insights

- More powerful learning is prompted when more of the five senses are engaged.
- Adequate time is needed for input, assimilation and output of new learning.
- Emotional well-being is essential to healthy intellectual functioning.

Brain Research Insights

- Our brain is a "social" brain.
- Learning involves conscious and unconscious processes.
- Complex learning is enhanced by challenge and inhibited by threat.
- Every brain is uniquely organized, with resulting differences of talent and preference.

Research indicates that students…

- need to see that they are learning something useful and relevant.
- abstract important concepts and are flexible in representing and using their knowledge when a subject is taught in multiple contexts with examples that apply what is being taught.
(Gick and Holyoak, 1983)

Research indicates that students…

- need mathematical models that apply to a whole variety of problems and to… create tools such as tables and graphs…
(NRC, 2000)
- learn if they are actively involved in choosing and evaluating strategies, considering assumptions, and receiving feedback.

Research indicates that students…

- learn by building on or transferring knowledge from previous experiences. They misinterpret new knowledge because they fail to make the connections with what they had previously learned.
- "…fail to connect everyday knowledge to subjects taught in school." (NRC, 2000)

Certain methods of teaching, particularly those that emphasize memorization as an end in itself, tend to produce knowledge that is seldom, if ever, used. Students who learn to solve problems by rote application of formulas, for example, often are unable to use their skills in new situations.

Development of intellectual competence requires more than the accumulation of discrete pieces of information.

(NRC, 2000)

Research about teaching suggests learning may be hindered by:

- too many topics too quickly.
- isolated sets of facts that are not organized and connected, or organizing principles without sufficient knowledge to make them meaningful.
- not understanding when, where, and why to use new knowledge.

Research about teaching suggests that learning by…

- "contrasting cases" helps learners notice new features they missed and identify important ones (e.g. square vs. rectangle vs. parallelogram).
- struggling at first with a concept enables students to benefit from a lecture that brings the ideas together.

(Schwartz and Bransford, 2000)

Implications for Teaching

- Teachers must draw out and work with the pre-existing understandings that their students bring with them.
- Teachers must teach some subject matter in depth, providing many examples in which the same concept is at work and providing a firm foundation of factual knowledge.
- The teaching of meta-cognitive skills should be integrated across the curriculum.

High quality learning occurs when there is…

- Learner independence and choice
- Intrinsic motivation and natural curiosity
- Rich, timely and usable feedback
- An occasion for reflection
- Active involvement in real-world tasks
- Cooperation with other learners
- A high-challenge, low-threat environment

Classroom teaching has often focused too narrowly on the memorization of information, giving short shrift to critical thinking, conceptual understanding, and in-depth knowledge of subject matter. The development of intellectual competence requires more than the accumulation of discrete pieces of information." (NRC, 2000)

What would help teachers?

Learning Environments that are…

- Learner-centered
- Knowledge-centered
- Assessment-centered
- Community-centered (NRC, 2000)

Having a coherent, connected pre-K – 12 curriculum focused on Mathematics

(Schmidt, et. al., 2002)

This stuff is not new!

- "Teachers open the door. You enter by yourself." –old Chinese proverb
- "When we have to learn to do, we learn by doing." –Aristotle
- "You cannot teach a man anything; you can only help him find it within himself." –Galileo

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