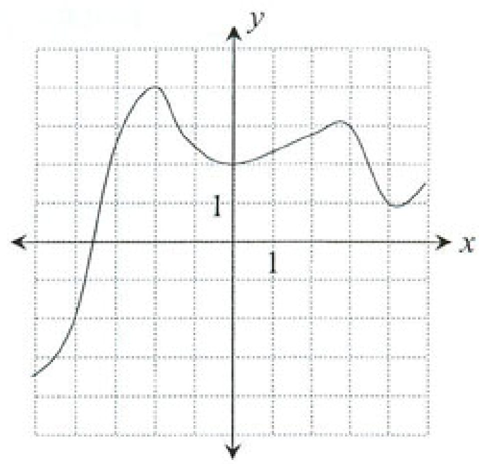
Name Key (59 points) **MA 131 Spring 2008** Chapter 1 Mini-exam

Student ID ______ *Wednesday, January 23, 2008* *Recitation Section* ______

Directions: *Do each of the following problems. Clearly indicate your final answer for each problem, and show all work required to obtain that answer. Questions that are worth only one point will have no partial credit possible. For questions worth more than one point, the correct answer is worth <u>only one point</u>. Whenever possible, final answers should be simplified.*

1. For this problem, the graph of $y = f(x)$ is shown to the right.
Each question is worth 1 point – 7 points total for this problem.



4 *a.* Write the value of $f(-2)$.

~ 2.3 *b.* *Estimate* the value of $f(1)$.

−4 *c.* For what value of x is $f(x) = -2$?

~ −4.5 *d.* *Estimate* the value of x for which $f(x) = -3$.

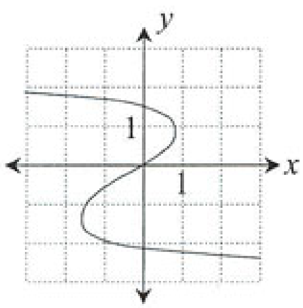
[−5, 5] *e.* Write the domain of f as drawn.

[−3.5, 4] *f.* Write the range of f as drawn.

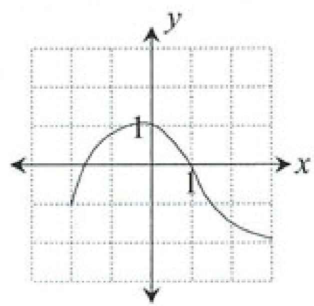
decreasing *g.* On the interval $3 \le x \le 4$, is the function increasing or decreasing?

2. Determine whether each graph as shown represents a function. *Each question is worth 1 point – 4 points total for this problem.*

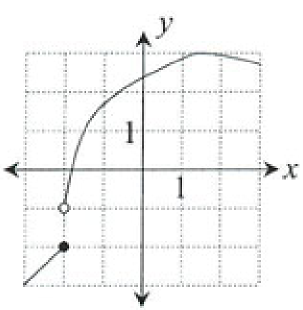
no *a.*



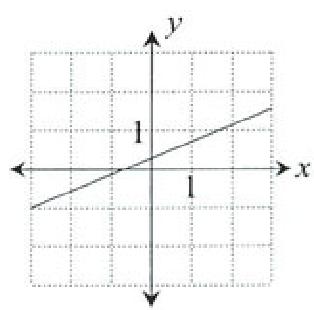
yes *c.*



yes *b.*



yes *d.*



11 points

3. Given the functions $f(x) = x^2 - 2x$ and $g(x) = x + 1$. *Worth 8 points total, distributed as indicated.*

15 *a.* Write the value of $f(-3)$. *[1 point]*

a^2-6a+8 *b.* Find an algebraic expression for $f(a-2)$. *[2 points]*

$(a-2)^2 - 2(a-2)$

$a^2 - 4a + 4 - 2a + 4$ $\qquad a^2 - 6a + 8$

0 *c.* Find the value of $(g \circ f)(1)$. *[2 points]*

$g(f(1)) = g(-1) = 0$

$x^2 - 1$ *d.* Write an algebraic expression for $(f \circ g)(x)$. *[3 points]*

$$
\begin{aligned}
f(g(x)) &= f(x+1) \\
&= (x+1)^2 - 2(x+1) \\
&= x^2 + 2x + 1 - 2x - 2 = x^2 - 1
\end{aligned}
$$

4. Determine whether the function $y = x^3 - 3x$ is odd, even, or neither. *[3 points]*

$f(x) = x^3 - 3x$

$f(-x) = (-x)^3 - 3(-x)$ $\qquad$ odd function

$= -x^3 + 3x = -(x^3 - 3x) = -f(x)$

5. Classify the following functions as a power function, root function, polynomial function, rational function, algebraic function, trigonometric function, exponential function, or logarithmic function. *Each part is worth 1 point – 5 points total for this problem.*

rational / algebraic *a.* $y = \dfrac{x^2+1}{x^3+x}$

trigonometric *b.* $y = \sin 2x$

polynomial/ algebraic *c.* $y = 3x^7 - 2x^5 + x^4 - 1$

exponential *d.* $y = 4^x$

power/ polynomial/ algebraic *e.* $y = x^4$

6. State the degree of the polynomial function $y = 5x^4 + 2x^3 - 3x + 9$. *[1 point]*

degree is 4

17 points

7. Suppose the "parent" function $f(x) = x^3$ is graphed on a coordinate plane. On the same set of axes, we then draw the graph of the transformed function $g(x) = (x-2)^3$. Describe the transformation that has been done on $f(x)$ to obtain $g(x)$. *[2 points]*

The original graph of f(x) has been shifted two units to the right (horizontal translation).

8. Solve each problem, writing your answer in the blank provided. *Each part is worth 3 points – 12 points total for this problem.*

__3__ *a.* Solve for x: $x = \log_5 125$.

$5^x = 125$ +2

$x = 3$ +1

__3__ *b.* Solve for x: $\log_3 x + \log_3(x-2) = 1$.

$\log_3 x(x-2) = 1$ +1

$3^1 = x(x-2)$ +1

$x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$ +1

$(x-3)(x+1) = 0$

$x = 3, \cancel{x = -1}$

__$(5, \infty)$__ *c.* Find the largest possible domain for the function $f(x) = \dfrac{1}{\sqrt{x-5}}$.

(or $5 < x$)

$\sqrt{x-5} \neq 0$ and $x - 5 \geq 0$ +1

$x > 5$

$x - 5 > 0$ +1

__$g^{-1}(x) = \frac{x+3}{2}$__ *d.* Given the function $g(x) = 2x - 3$, find its inverse $g^{-1}(x)$.

$y = 2x - 3$ +1

$x = 2y - 3$ +1

$x + 3 = 2y$

$\frac{x+3}{2} = y$ +1

9. Find the exact value of each expression. *Each part is worth 2 points – 6 points total for this problem.*

__8__ *a.* $e^{3\ln 2}$

$e^{\ln 2^3} = e^{\ln 8} = 8$

__2__ *b.* $\log_6 12 + \log_6 3$

$\log_6 (12)(3) = \log_6 36 = 2$

__$-1/2$__ *c.* $\cos\left(\frac{2\pi}{3}\right)$

$\frac{2\pi}{3} = 120°$

$-\cos\frac{\pi}{3} = -1/2$

20 points

10. As referenced in class, suppose you take a sheet of paper and fold it in half. Then, you fold those layers in half again, and repeat this process many more times; each time you fold the entire "stack" of paper in half. *We will assume for purposes of this problem that any physical limitations on folding thick stacks of paper are unimportant.*

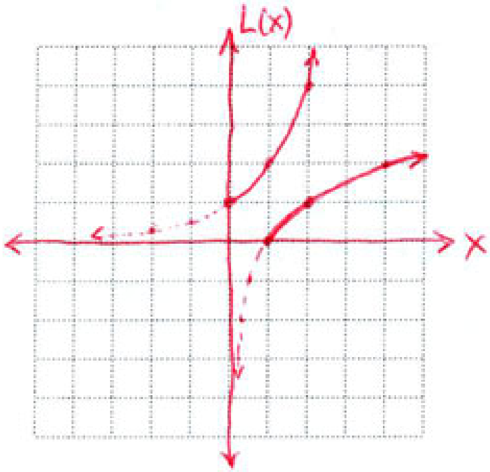
a. Complete the table to the right showing the data resulting from this process. *[3 points]*

Number of Folds (x)	Layers of Paper (L)
0	1
1	2
2	4
3	8
4	16
5	32
6	64
7	128
8	256
9	512

b. If we let L represent the number of layers of paper, and x represents the number of folds made, write an equation that mathematically describes L as a function of x. *(i.e. $L(x) = \ldots$) [2 points]*

$L(x) = 2^x$ "exponential growth"

c. On the grid printed below, draw a sketch showing the relationship between number of folds x and number of layers L.
Be sure to properly label your graph – also, note that you will be drawing another graph on this same grid in part e. [2 points]



Data only yields the solid part of the graph.

The lighter part of the graph comes from the equation <u>only</u> → $L(x) = 2^x$.

d. Using your equation from part *b*, find the equation that represents the inverse $L^{-1}(x)$. *[2 points]*

$L(x) = 2^x$

$y = 2^x$

$x = 2^y$

$y = \log_2 x$

$L^{-1}(x) = \log_2 x$

e. On the same grid provided above, sketch the inverse you found in part *d.* *[2 points]*

(switch all (x, y) pairs)

11 points