MA 131 Calculus I
Sample Problems – Test 1 *(February 27, 2008)*

1. You need to be able to correctly evaluate a variety of limits, from techniques discussed in chapter 2. Here are a few samples…

   a. $\lim_{x\to -4} \dfrac{\sqrt{x^2+9}-5}{x+4}$

   b. $\lim_{x\to -1} \dfrac{x^2-1}{2x-2}$

   c. $\lim_{h\to \infty} \dfrac{\sqrt{9h^6-h}}{h^3+1}$

   d. $\lim_{x\to 1^+} \dfrac{x^2-9}{x^2+2x-3}$

   e. $\lim_{u\to 1} \dfrac{u^4-1}{u^3+5u^2-6u}$

2. You need to be able to correctly find a variety of derivatives using the differentiation rules we have discussed in chapter 3. You also need to be able to use the definition of derivative as we have used it in class: $f'(x)=\lim_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$. If evaluating the limit at a specific value of $x = a$, instead of the general derivative function, we have used $f'(a)=\lim_{h\to 0}\dfrac{f(a+h)-f(a)}{h}$. Here are some examples:

   a. $y=\dfrac{3t-5t^2\sqrt{t}}{\sqrt{t}}$

   b. $f(x)=5e^{2x}\csc(x)$

   c. $g(x)=\dfrac{x^2-x}{3x^2+5}$

   d. $y=3\sin\left(\dfrac{5\pi}{6}\right)$

   e. $x^3+y^3=3$ (*use implicit differentiation*)

   f. $f(x)=\dfrac{3}{x}$ (*use the definition of derivative for this*)

3. You should understand the concept of continuity well, and be able to determine places on a function where there are discontinuities. For example, consider the function defined in pieces as follows:

$$f(x)=\begin{cases}\dfrac{2x^2-5x-3}{x^2-4x+3} & \text{if } x<3\\ mx+4 & \text{if } x\geq 3\end{cases}$$

Determine the location(s) and type(s) of any discontinuities of this function. Also, find the value of *m* that will make the function continuous at $x = 3$.

4. You should be able to successfully complete word problems that relate to the derivative, since derivatives represent rates of change. A couple such questions are included here for practice.

a. Given the function $y = \dfrac{2x}{(x+1)^2}$, write an equation of the line tangent to this graph at the point where $x = 1$.

b. A tank holding 36 gallons of water has a drain at the bottom which is opened at time $t = 0$. The volume of water in the tank at time t minutes, with $0 \le t \le 3$, is $V(t) = 36 - 24t + 4t^2$ gallons.

Note: Include units in your answers and be careful with signs.

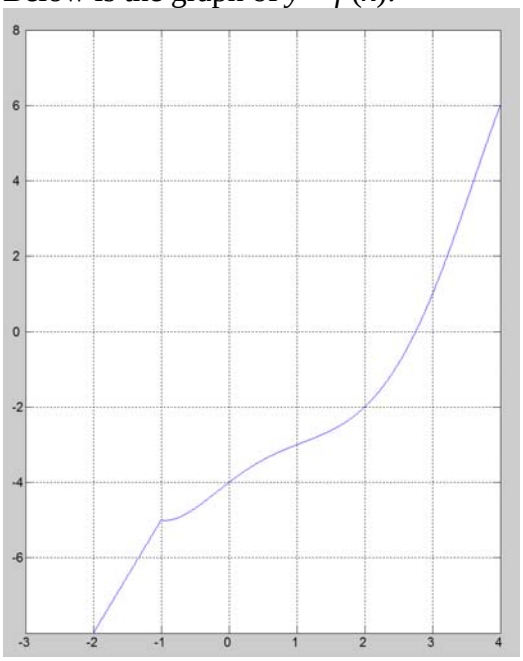
i. What is the instantaneous rate at which water is draining out of the tank at time $t = 2$ minutes?

ii. How much water drains out of the tank in the first two minutes?

iii. What is the average rate at which the water drains out over the first two minutes?

5. You need to be able to make interpretations about a function and its derivative from the graph, as illustrated in the following sample.

Below is the graph of $y = f(x)$:



a. Determine any locations where $f(x)$ is <u>not</u> differentiable.

b. On the axes provided (*next page*), sketch each of the following:

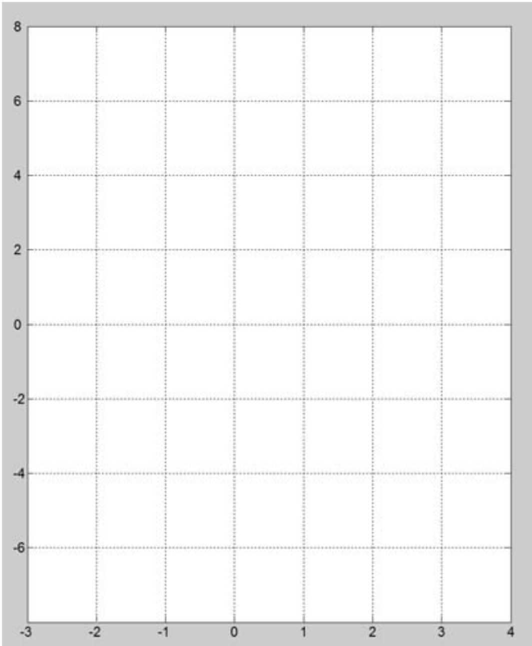
i. $g(x) = f(x - 1)$

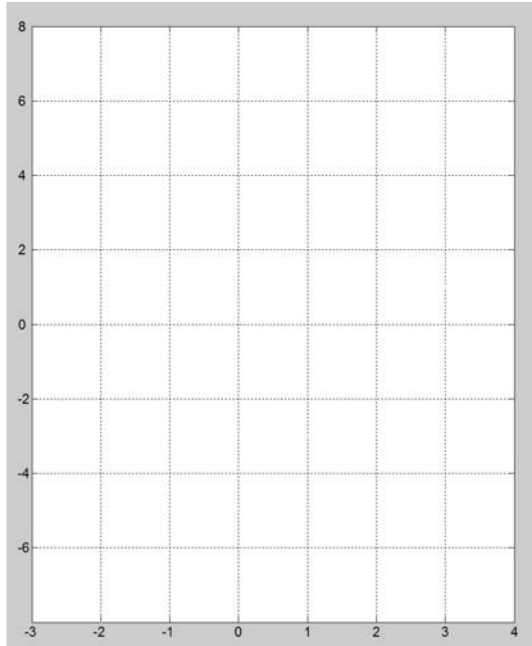
ii. $h(x) = f(-x)$

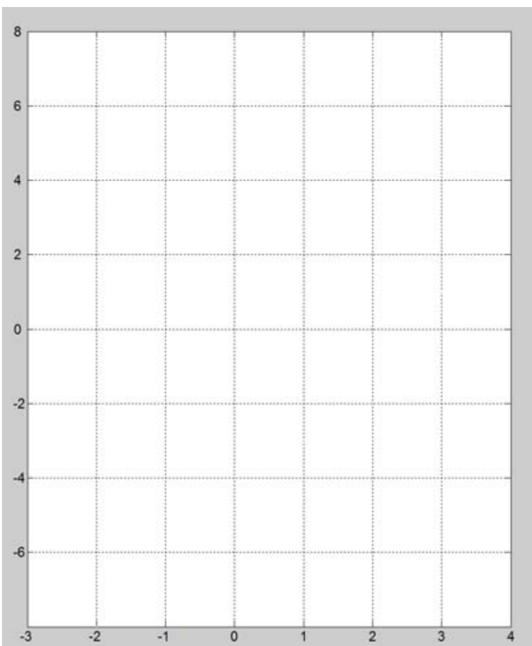
iii. $k(x) = f(2x)$

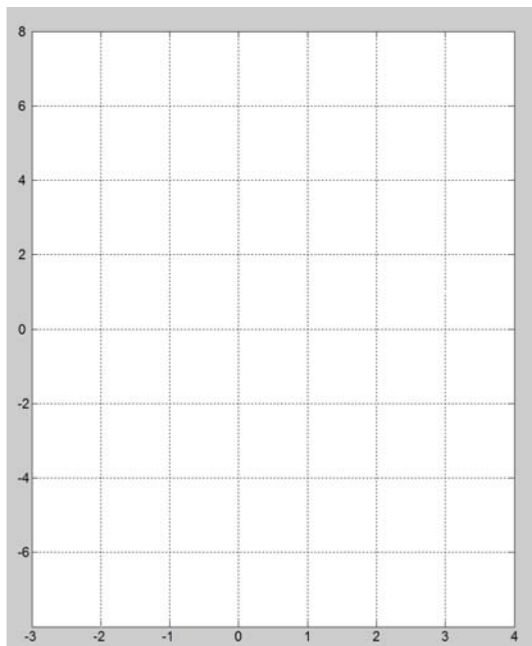
iv. the derivative $f'(x)$

c. Find the value of $f^{-1}(-2)$









Also, recall that trig functions, especially the inverse trig functions, were not tested on the mini-chapter 1 exam, so make sure that you know how to deal with trigonometry well! Study the identities that exist between the trig functions, as given on a recent handout.