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In this second article in the series, the author addresses the professional development standard "Knowing Mathematics Pedagogy" (pp. 151–59). As in the article in the November 1992 issue, teachers are urged to reexamine what happens in their classrooms. However, the emphasis in this article is on the effectiveness of teachers' classroom practices. Are our typical teaching strategies effective in helping students communicate, apply mathematics, reason, and solve problems? Teachers are challenged to take a risk and are given suggestions to meet the challenge.—*Ed.*

Rethinking How We Teach:
Learning Mathematical Pedagogy

A human child is at the most learning age of the most learning species that has yet evolved on this planet" (Skemp 1987, p. 101). So begins Skemp's essay titled "A New Model of Intelligence." Skemp goes on to point out that the extent to which the learner's potential will be realized is largely dependent on the individual's teachers. In Skemp's view, teaching is any kind of action that influences the learning process. This "learning process" is not directly observed but, warns Skemp (1987), "a person who intervenes without an adequate mental image of what is going on inside is as likely to do harm as good" (p. 101).

The writers of the *Professional Standards for Teaching Mathematics* (NCTM 1991) recognized the validity of this view. As professionals we must develop our knowledge to evaluate our teaching and better assess understanding on the part of students. Standard 4 in the section "Standards for the Professional Development of Teachers of Mathematics" addresses the teachers' need to learn how "they can help their students come to understand and be able to use mathematics" (NCTM 1991, 151). Other points in Standard 4 include the teachers' need to use instructional materials and technology to represent different concepts and procedures, to choose among instructional strategies and classroom organizational models, to encourage students' discourse, and to assess students' understanding. In short, Standard 4 acknowledges that we must know mathematical pedagogy and, more importantly, must change our teaching to reflect this new knowledge.

So what do we do? Many of us studied both our mathematics and our pedagogy some time ago. We probably were not taught a constructivist manner and, therefore, have not taught from this perspective. (Constructivists believe that students construct their own meanings of mathematical concepts and procedures when they are given the opportunity to become actively involved in learning. Teachers facilitate students' active construction rather than dispense rules and algorithms.) Even if we just recently began to teach, the chances are high that much of what is now being proposed is new, different, and unsettling. Using cooperative groups in class may not be part of our experience, having students carry on discussions is probably disturbing, and integrating technology is probably uncomfortable. Finally, we recognize that the new approaches require different assessments, which may seem particularly burdensome.

How do you start to make changes?

You profess to believe and subscribe to a set of practices that will help your students to learn mathematics. You have a theory of teaching, which was shaped by the way you were taught, the way you were taught to teach, and classroom experiences. You have been successful—when measured by grades, SAT/ACT scores, and former students' success in their mathematics courses that followed. It is precisely because of that success that change may be difficult!

You read and know that the SAT/ACT tests are changing, that college freshman courses are being restructured, that graphing calculators are now

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Collect and share classroom teaching practices

used in ninth-grade algebra as well as upper high school and college courses, and that what we know about learning—the constructivist view—requires new approaches in the classroom. As a reader of this journal, you are aware of the reforms recommended in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989) and you know that not all that you do agrees with research findings on teaching and learning. Perhaps you tend to use few manipulatives, still teach "whole groups," and have not integrated technology into your classes. You have decided to make some changes. So how do you start? What do you do? Why not return to the three Rs—reflect, risk, and revise?

REFLECT

Any attempt to reform one's teaching must begin with reflection. Freudenthal (1973) counsels that teachers must learn from "one's own and others' examples to analyze the instruction one is attempting to give, is giving, and has been giving" (p. 167). "In my experience" he writes, "there are not many teachers who consider their own teaching and that of others as a matter for investigation . . ." (pp. 167–68).

Take time to jot down what you believe and what you do—daily, weekly, every semester. Do you turn to last year's notes and tests as a matter of course, and why? Why do you teach ratio the way you do? Why do you teach geometry as you do? Think about the classes you taught yesterday. How and why were they organized? What beliefs do you hold that influenced your choice of classroom activities? Define your theory of teaching as clearly as possible.

Shulman (1987) has urged us to think deeply about our teaching—what do we do and why do we do it. He recommended that we collect the best of our classroom creations and share these with colleagues. However, Shulman's "wisdom of practice" must not be confused with the limitations of unexamined practice. Simply to record what we do without analysis will not serve us well.

Examining our beliefs must be done in the context of what is now known about teaching and learning. It may have been the case, as Skemp (1987) notes, that we once had little insight of what was going on in the learning process and perhaps we still do. But what is now known far outdistances what we knew just a short time ago (see Grouws [1992]). To reflect on what we do without referring to this rich knowledge is to risk examination in a vacuum. Thus, as you reflect on your teaching, plan reading that is large in scope.

I recommend *The Psychology of Learning Mathematics* (Skemp 1987). Chapter 12, "Relational Understanding and Instrumental Understanding," is an excellent place to start. (I seldom read from page 1 to the end!) Next, the 1990 Yearbook of the National Council of Teachers of Mathematics, *Teaching and Learning Mathematics in the 1990s* (Cooney 1990), will supply insights into constructivism, effective models of teaching, reasons for proposed change in assessment, and the role technology must play. As you read, contrast what is being proposed with what you do. Compare approaches, intent, and outcomes. Read *Constructivist Views on the Teaching and Learning of Mathematics* (Davis, Maher, and Noddings 1990). This monograph furnishes additional background on constructivism, descriptions of mathematical activity engaged in by children, and ways that teachers can structure classes to promote this activity. Read *Connecting Mathematics* (Froelich 1991), one of the volumes in the Addenda Series for grades 9–12. Review the section "Reshaping Pedagogy" (p. vi) and ask yourself if you have moved in the direction suggested in the reading.

Read "The Learning of Mathematics: Stepping Stones to Planning" in *Secondary Mathematics Instruction: An Integrated Approach* (Farrell and Farmer 1988). This excellent overview may help you better define or sharpen those beliefs that guide your teaching and allow you to decide what approaches should change. Examine the tests you give. Compare and contrast ways in which you assess students with those recommended in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), the 1990 Yearbook, and the recently released *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark 1991). Finally, study the several vignettes that relate to Standard 4 in the *Professional Standards for Teaching Mathematics* (p. 159). How comfortable are you with the teaching situations described in the book? How closely do they reflect what you do?

RISK

A major tenet of constructivism is that new learning must be integrated into one's existing schemas. If reading and other activities have created "disequilibration," then you are ready to act. Farrell and Farmer wrote that equilibration is the tendency of the mind to "adjust and restructure in response to apparent aberrations" (1988, 57). The aberration in this situation is the discrepancy between methods you are using and those you keep reading, hearing, and perhaps seeing. To attempt to change is a risk. The lesson may fall flat, the students may "object," or the new ways may cause you to be far less effective than you believe you should be. To wait for students' answers, to trust learners to struggle and make sense of a situation—given that you are assured that the necessary schemas

they need are in place, or to refuse to be the final arbiter and to insist on logic and mathematical reasoning as verification of a solution may require great discipline on your part. The rewards of change, however, are truly wonderful.

Armed with your many new insights, attend workshops and courses that will expand your knowledge and that will offer opportunities to hear what others are saying and how they are interpreting the recommendations for change. Try to identify the workshops in which leaders truly model active learning, not just talk about it. Not all workshops are created equal! Ask questions of the presenter and inquire about the problems that were encountered when the activities were tried with students. When you return, try some of the suggestions and record the students' reactions, your feelings, and which approaches you would change the next time. Review both the workshop and your teaching in light of the reading you've done. Do you see congruence?

Another rich source of help may be faculty and programs at a local college or university. Contact the mathematics department or mathematics education department to find courses, seminars, and other activities that may be of help. Look into curriculum development or other mathematics education projects or summer opportunities in which you could participate.

If you have colleagues at the college, encourage them to apply for teacher enhancement grants or other assistance from the National Science Foundation and the Department of Education. Work with one professor at the university to become a teacher-partner in research being done on mathematics teaching and learning. Many of our college colleagues are anxious to work with us, so make the first move and find out how you can work together to the mutual benefit of instructors as well as students.

Plan a local staff-development activity around particular articles from the *Mathematics Teacher,* such as "Cooperative Learning Works in Mathematics" (Sutton 1992) or any of the myriad fine articles that have appeared recently. Use this in-service education time to ask if colleagues have changed their instructional patterns and why. Work with colleagues to try a cooperative-learning lesson. Then plan a second meeting to share successes and failures. Learn from the failures, then go back and try another activity.

Plan a lesson with specific goals in mind. Perhaps you want to improve student discourse. Either audiotape or videotape the class. Review the tape and analyze what happened. Were the students involved as much as you wanted? How did the discussion flow? Team up with a colleague, share the tape, discuss what activity is seen, and contrast your review and analysis with that of your colleague.

Risk changing other factors that influence the way you teach. For example, desks in rows do not usually encourage the teaching and learning recommended by current research. Rows are associated with old models of instructional practice—models that students recognize and with which they are comfortable. After summer-program study where cooperative groups, paired problem solving, journal writing, project work, and using technology had been modeled, many participants found that the first step to begin incorporating these practices in their own teaching was to change the physical setup of the classroom. These teachers reasoned that if old instructional models are removed, students may accept new practices more readily.

One teacher, Alan, has rearranged his room as shown in **figure 1.** The horseshoe pattern encourages discourse among *all* students and the teacher, not just between *a* student and the teacher. The table in the middle of the horseshoe can hold students' journals, materials to be used that day, or student resources. This configuration allows for easy pairing of students, permits groups of students to work on a project, and has radically changed all five of Alan's classes. Alan finds that he is "closer" to individuals as well as groups of students. The arrangement encourages more interactive presentations of new material—short expositions followed by discussions and questions, presentation of problems that allow students to search for patterns and form their own generalizations, and debates among students about the nature of a problem while the teacher serves as moderator. The setup of the class

Reflect on what you believe about classroom activities

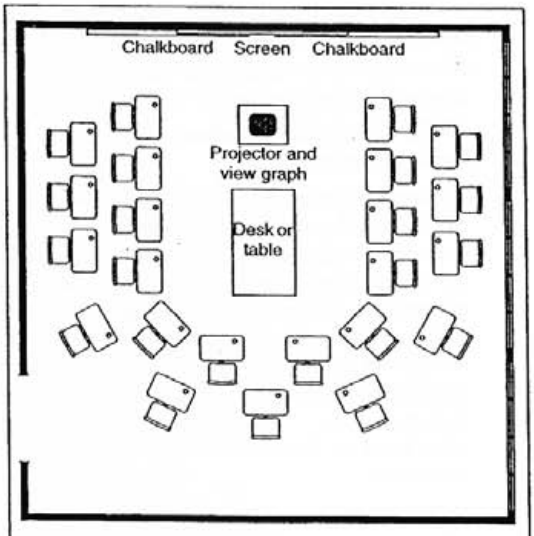

Fig. 1
This classroom arrangement promotes discourse among all students.

encourages students' participation, and an observer quickly "feels" the excitement in the class.

Many other physical arrangements will work; experiment to find what works for you. However, unless you change the sociology of the class you may find that students will subtly guide you back to old and comfortable patterns. Secondary school students, in particular, have developed strong beliefs about what a mathematics lesson should be and will try to steer you toward those ways. New situations cause discomfort, so students and instructors alike will seek to return to their "comfort zones" (McWilliams and McWilliams 1991, 25–33).

REVISE

Not everything we try or every lesson we teach is successful. As we break with the old ways and attempt to use new ways, don't expect perfection. When working with either computers or calculators, for example, you have quickly learned that these tools present both opportunities and problems. Students respond to using technology, but how do you assure that what is going on at individual desks is related to the lesson? Review your organization of those lessons, plan discussions with colleagues, and structure new lessons. This process will unveil effective ways that you can record, share, and further revise as your expertise grows. Remember Shulman's (1987) charge to record your reasons for doing what you did. Would a different arrangement of the room help? Does being in the center of the action assure that each pair of students is working on the assigned work rather than exploring new functions?

Students may steer you back to the old ways

Researchers have compared expert and novice teachers. The findings may help you evaluate your efforts and review and revise lessons with the aim of attaining "expert status." Koehler and Grouws (1992, 121), in their summary of expert-novice research, reported that

> [e]xpert mathematics teachers weave a series of lessons together to form an instructional topic in ways that consistently build upon and advance material introduced in prior lessons. Experts also construct lessons that display a highly efficient internal structure, one that is characterized by fluid movement from one type of activity to another, by minimal student confusion during instruction, and by a transparent system of goals. . . . Novice teachers' lessons, on the other hand are characterized by fragmented lesson structures. . . .

The combination of your study and reading, attendance at workshops or local staff-development activities, collaboration with researchers and curriculum developers, and trials and revisions will soon result in pedagogical changes—changes that will improve students' learning.

CONCLUSION

As McWilliams and McWilliams (1991) point out, we are wedded to our comfort zones, as are our supervisors and colleagues. Thus, any attempt at change is discomforting for us and those around us. We can easily excuse inaction by referring to a lack of materials—I have only one calculator, the absence of funding, or the absence of support. Small steps can be taken to make significant change in our classrooms. The matters of discourse, applications, students' activity, and assessment are but a few areas that are within our control. Remember, all who attempt to change are faced with similar hurdles, so you are not alone.

Borko and her colleagues (1992, 221) recently noted the following:

> It will never be possible, within the constraints of a single mathematics methods course or even an entire preservice teacher preparation program, to enable prospective teachers to learn all that they need to know and believe about mathematics and mathematics pedagogy in order to teach effectively.
>
> In the constructivist tradition, we must reflect on current knowledge about learning, examine our own teaching, and identify the discrepancies that exist between our methods and those we know we should be using. We are well aware that as novices or experts we can still do much to improve our teaching and that the responsibility for improvement lies with us. We must do the learning, and we must reconstruct our own view of teaching.

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