

1. Definition of derivative: $f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$

$$f'(1) = \lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{3(1+h)+2} - \frac{1}{5}}{h} = \lim_{h \rightarrow 0} \frac{\frac{1}{3h+5} - \frac{1}{5}}{h} = \lim_{h \rightarrow 0} \frac{\frac{5-3h-5}{5(3h+5)}}{h} = \lim_{h \rightarrow 0} \frac{-3h}{5(3h+5)(h)},$$

$$\text{which is just } \lim_{h \rightarrow 0} \frac{-3}{5(3h+5)} = -\frac{3}{25}.$$

You can always check this by doing a regular derivative and plugging in 1.

2. $y = \cos^{-1} x \Rightarrow \frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$. *Know your derivatives!*

3. $h(x) = e^{2x^3} \Rightarrow h'(x) = (e^{2x^3})(6x^2) = 6x^2 e^{2x^3}$. *Know the chain rule!*

4. $G(x) = \int_{-2}^x \frac{t}{\sqrt[3]{6+t^4}} dt \Rightarrow G'(x) = \frac{x}{\sqrt[3]{6+x^4}}$. *We applied the Fundamental Theorem (part 1).*

5. $z = \tan^2 \theta \Rightarrow \frac{dz}{dt} = 2(\tan \theta)^1 (\sec^2 \theta) = 2 \tan \theta \sec^2 \theta$. *Chain rule, again.*

6. $f(t) = \csc t \Rightarrow f'(t) = -\csc t \cot t$.

7. $f(t) = t^2 \ln(t) \Rightarrow f'(t) = t^2 \left(\frac{1}{t}\right) + (2t)(\ln t) = t + 2t \ln t$. *We used the product rule.*

8. $y = \frac{x^2 - 5}{x^2 + 4} \Rightarrow \frac{dy}{dx} = \frac{(x^2 + 4)(2x) - (x^2 - 5)(2x)}{(x^2 + 4)^2} = \frac{2x^3 + 8x - 2x^3 + 10x}{(x^2 + 4)^2} = \frac{18x}{(x^2 + 4)^2}$. *We used the quotient rule.*

9. $g(\theta) = \cos^4(e^\theta) = (\cos(e^\theta))^4 \Rightarrow 4(\cos(e^\theta))^3 (-\sin(e^\theta))(e^\theta) = -4e^\theta \sin(e^\theta) \cos^3(e^\theta)$. *This problem required two applications of the Chain Rule.*

10. $y = (t+2)^{\ln(t+2)} \Rightarrow \ln y = \ln((t+2)^{\ln(t+2)}) \Rightarrow \ln y = \ln(t+2)(\ln(t+2)) \Rightarrow \ln y = (\ln(t+2))^2$. Now we can take the derivative implicitly: $\frac{1}{y} \frac{dy}{dt} = 2(\ln(t+2)) \left(\frac{1}{t+2}\right) \Rightarrow \frac{dy}{dt} = \frac{2y \ln(t+2)}{t+2}$. But, this contains a y, so we substitute back: $\frac{dy}{dt} = \frac{2(t+2)^{\ln(t+2)} \ln(t+2)}{t+2}$. *We could have some alternative forms for this, but that is really not necessary.*

Check back for more solutions soon!