

14. a.  $\lim_{x \rightarrow 0} \frac{x^2}{\cos x} = 0$  This one's just a plug-and-chug.

b.  $\lim_{x \rightarrow 0} \frac{\sin^{-1}(3x)}{4x} = \lim_{x \rightarrow 0} \frac{\frac{1}{\sqrt{1-(3x)^2}}(3)}{4} = \frac{3}{4}$  This one uses l'Hôpital's Rule.

c.  $\lim_{x \rightarrow 4} \frac{x^2 - 2x - 8}{x^2 - 16} = \lim_{x \rightarrow 4} \frac{(x-4)(x+2)}{(x-4)(x+4)} = \lim_{x \rightarrow 4} \frac{x+2}{x+4} = \frac{6}{8} = \frac{3}{4}$  The ol' "factor and cancel" trick!

d.  $\lim_{x \rightarrow -3} \left( \frac{5 - \sqrt{x^2 + 16}}{x + 3} \right) \left( \frac{5 + \sqrt{x^2 + 16}}{5 + \sqrt{x^2 + 16}} \right) = \lim_{x \rightarrow -3} \frac{25 - (x^2 + 16)}{(x + 3)(5 + \sqrt{x^2 + 16})} = \lim_{x \rightarrow -3} \frac{9 - x^2}{(x + 3)(5 + \sqrt{x^2 + 16})}$ , and we can now factor the numerator:  $\lim_{x \rightarrow -3} \frac{(3-x)(3+x)}{(x+3)(5+\sqrt{x^2+16})} = \lim_{x \rightarrow -3} \frac{3-x}{5+\sqrt{x^2+16}} = \frac{6}{10} = \frac{3}{5}$ .

14. (On page 2)  $v(t) = 6t^2 + 6t + 1$

a. Average acceleration is  $\bar{a} = \frac{v(2) - v(0)}{2 - 0} = \frac{37}{2} = 18 \frac{1}{2} \frac{\text{cm}}{\text{sec}}$ .

b. Instantaneous acceleration requires the derivative:  $a(t) = \frac{dv}{dt} = 12t + 6$ , so  $a(2) = 30$  cm per second.

c. Total displacement is an integral:  $\int_0^2 (6t^2 + 6t + 1) dt = (2t^3 + 3t^2 + t) \Big|_0^2 = 16 + 12 + 2 = 30$  cm.

15.  $f(x) = 2 \sin x + 2 \cos x$ , so  $f'(x) = 2 \cos x - 2 \sin x$ . Setting this to zero, we find critical numbers:

$2 \cos x - 2 \sin x = 0 \Rightarrow \cos x = \sin x$ . The only place this happens between 0 and  $\frac{\pi}{3}$  is at  $\frac{\pi}{4}$ , so that is our only critical number. Then, we test the two endpoints of the interval, and the critical number, back in

the original.  $f(0) = 2 \sin 0 + 2 \cos 0 = 2$ , while  $f\left(\frac{\pi}{3}\right) = 2 \sin\left(\frac{\pi}{3}\right) + 2 \cos\left(\frac{\pi}{3}\right) = 2\left(\frac{\sqrt{3}}{2}\right) + 2\left(\frac{1}{2}\right) = \sqrt{3} + 1$ .

Finally,  $f\left(\frac{\pi}{4}\right) = 2 \sin\left(\frac{\pi}{4}\right) + 2 \cos\left(\frac{\pi}{4}\right) = 2\left(\frac{\sqrt{2}}{2}\right) + 2\left(\frac{\sqrt{2}}{2}\right)$ , which is  $2\sqrt{2}$ . Since we are given that

$\sqrt{2} \approx 1.414$ ,  $2\sqrt{2} \approx 2.828$ , and since  $\sqrt{3} \approx 1.732$ ,  $\sqrt{3} + 1 \approx 2.732$ , and we can compare these. So our absolute min is  $(0, 2)$ , and the absolute max is  $\left(\frac{\pi}{4}, 2\sqrt{2}\right)$ .