16. $f(x) = x^3 - 3x^2 + 5$

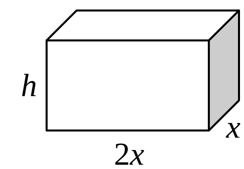
a. $f'(x) = 3x^2 - 6x$, so $f'(x) = 0 \Rightarrow 3x^2 - 6x = 0 \Rightarrow 3x(x-2) = 0 \Rightarrow x = 0,\ 2$, which are the critical numbers. So, we put these on a number line, and test, for example, –1, 1, and 3 in the derivative. This gives us +, –, + respectively, so the function is increasing on $(-\infty, 0) \cup (2, \infty)$ and decreasing on $(0, 2)$.

b. From the work in part *a*, clearly there is a local max at *x* = 0 and a local min at *x* = 2. To find the *y*-values, we plug into the original function: (0, 5) is a local max and (2, 1) is a local min.

c. $f''(x) = 6x - 6$, so $f''(x) = 0 \Rightarrow 6x - 6 = 0 \Rightarrow x = 1$. Making a number line and checking 0 and 2 (for instance) in the second derivative, we get – then +. So, $f(x)$ is concave down on $(-\infty, 1)$ and concave up on $(1, \infty)$.

d. From work done in part *c*, there is obviously an inflection point at *x* = 1. The coordinates of this point are (1, 3).

17. A diagram is shown to the right. I establish the following variables for the problem:



x = width of the base of the box
2*x* = length of the base of the box
h = height of the sides of the box

$V = lwh$, so $10 = 2x(x)(h) \Rightarrow h = \frac{10}{2x^2} = \frac{5}{x^2}$

We want to minimize cost, so we have to create a cost function:

$$C = \$10[x(2x)] + 4(\$6[h(2x)]) \Rightarrow C = 20x^2 + 48hx \Rightarrow C(x) = 20x^2 + 48\left(\frac{5}{x^2}\right)x \Rightarrow C(x) = 20x^2 + \frac{240}{x}.$$

Note that this total cost comes from the cost of the base (only one) and the sides (there are four).

So, let's use Calculus to minimize:

$$C'(x) = 40x - \frac{240}{x^2}, \text{ so } C'(x) = 0 \Rightarrow 40x - \frac{240}{x^2} = 0 \Rightarrow 40x = \frac{240}{x^2} \Rightarrow 40x^3 = 240 \Rightarrow x^3 = 6 \Rightarrow x = \sqrt[3]{6}.$$

Now, if I make a first derivative number line and check *x* = 1 and *x* = 2 in the first derivative, I get – then +, so clearly $x = \sqrt[3]{6}$ is a min. Thus, the dimensions of the container are $\sqrt[3]{6}$ meters wide, $2\sqrt[3]{6}$ meters long, and $\frac{5}{\sqrt[3]{6^2}} = \frac{5\sqrt[3]{6}}{6}$ meters high.

18. Use implicit differentiation: $x^3 \frac{dy}{dx} + 3x^2 y - \left(x(2y)\frac{dy}{dx} + y^2\right) = 0$. *Note that we used the product rule twice.* Now I distribute the negative: $x^3 \frac{dy}{dx} + 3x^2 y - 2xy\frac{dy}{dx} - y^2 = 0$. Collecting "like terms" gives us $\frac{dy}{dx}(x^3 - 2xy) = y^2 - 3x^2 y \Rightarrow \frac{dy}{dx} = \frac{y^2 - 3x^2 y}{x^3 - 2xy}$. So, to find the slope of the tangent line at (2, 2), we simply plug in *x* = 2 and *y* = 2: $\frac{dy}{dx} = \frac{2^2 - 3(2^2)(2)}{2^3 - 2(2)(2)} = \frac{4 - 24}{8 - 8} = \frac{-20}{0}$, so the slope is undefined. (This means the tangent line is vertical.)

19. $f(x)=\sqrt[4]{x}$, so $f(x)=x^{1/4} \Rightarrow f'(x)=\frac{1}{4}x^{-3/4}$. Since a = 16, $f(a)=2$ and $f'(a)=\frac{1}{4(16)^{3/4}}=\frac{1}{32}$. Now, the local linearization formula is $L(x)=f(a)+f'(a)(x-a)$, so $L(x)=2+\frac{1}{32}(x-16)$. (We could modify this to a linear expression: $L(x)=\frac{1}{32}x+\frac{3}{2}$, if that's all that was requested by the problem.) Now, to approximate $\sqrt[4]{15.52}$ to *two decimal places*, we simply use x = 15.52. So, $f(x)\approx L(x)$, and $L(15.52)=2+\frac{1}{32}(15.52-16)=2+\frac{1}{32}(-0.48)=2+\frac{1}{32}\left(\frac{-48}{100}\right)=2-\frac{3}{200}=2-\frac{15}{1000}=1.985$. So, to two decimal places, our answer (the approximation) is 1.99. *Note that the actual fourth root of 15.52 rounds to 1.98 to two decimal places.*

20. $A=\pi r^2$, so $\frac{dA}{dt}=2\pi r\frac{dr}{dt}$. We are given that $\frac{dr}{dt}=11\frac{\text{cm}}{\text{sec}}$ in the problem itself, and we know that after 3 seconds, the radius will be 33 cm. So, $\frac{dA}{dt}=2\pi r\frac{dr}{dt}=2\pi(33)(11)=726\pi\frac{\text{cm}^2}{\text{sec}}$.