21. $\int \frac{1}{3t+4}\,dt$ *Use substitution.*

Let $u = 3t + 4$, so $du = 3dt$, and $\frac{du}{3} = dt$. So, the integral becomes $\frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C = \frac{1}{3}\ln|3t+4| + C$.

22. $\int \left(\cos^5(x)\right)\left(\sin x\right)\,dx$ *Use substitution.*

Let $u = \cos x$, so $du = -\sin x\,dx$, which means $-du = \sin x\,dx$. Then the integral is $\int u^5(-du) = -\int u^5\,du$ $= -\frac{u^6}{6} + C = -\frac{1}{6}\cos^6(x) + C$.

23. $\int e^{-2t}\,dt$ *Use substitution.*

Let $u = -2t$, so $du = -2dt$ and $\frac{du}{-2} = dt$. This gives $\int e^u\left(\frac{du}{-2}\right) = -\frac{1}{2}\int e^u\,du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-2t} + C$.

24. $\int_0^2 \left(x^3 - 3\right)dx$ *Use power rule, then evaluate.*

$$\int_0^2 \left(x^3-3\right)dx = \left(\frac{x^4}{4} - 3x\right)\Bigg|_0^2 = \frac{2^4}{4} - 3(2) - \left(\frac{0^4}{4} - 3(0)\right) = 4 - 6 = -2.$$

25. $\int x\sqrt{3x+1}\,dx$ *Use substitution.*

Let $u = 3x + 1$, so $du = 3dx$, or $\frac{du}{3} = dx$. Also, we know that $u - 1 = 3x \Rightarrow \frac{u-1}{3} = x$. So, we get the new integral: $\int\left(\frac{u-1}{3}\right)\sqrt{u}\,\frac{du}{3} = \frac{1}{9}\int (u-1)u^{1/2}\,du = \frac{1}{9}\int\left(u^{3/2} - u^{1/2}\right)du = \frac{1}{9}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C$.

Simplifying, distributing, and re-substituting gives us $\frac{2}{45}\sqrt{(3x+1)^5} - \frac{2}{27}\sqrt{(3x+1)^3} + C$.

26. $\int_1^e \frac{\sqrt{\ln x}}{x}\,dx$ *Use substitution – this seems to be a theme…*

Let $u = \ln x$, so $du = \frac{1}{x}dx$. Also, if $x = 1$, then $u = 0$, and if $x = e$, then $u = 1$. So, our new definite integral becomes $\int_0^1 \sqrt{u}\,du = \int_0^1 u^{1/2}\,du = \frac{u^{3/2}}{3/2}\Bigg|_0^1 = \frac{2}{3}(1-0) = \frac{2}{3}$.