Homework 1 Problem 9 Solution

a. By using the (unrestricted) growth estimate of 1.7% from the 1990s, and assuming it has occurred, we can calculate the following (first, we look at the general population growth model, which is drawn out in more detail in your text):

Suppose P represents the population at any time t, it is reasonable to suppose that the rate of change of population is proportional to the population itself (more people, more "making of people"). Thus, $\frac{dP}{dt} = kP$, so $\frac{dP}{P} = k\ dt$, which we can solve by integrating both sides to obtain $\int \frac{1}{P}\ dP = \int k\ dt$, so $\ln P = kt + C$, or $e^{\ln P} = e^{kt+C}$, which becomes $P = e^C e^{kt}$. If we simply call e^C the initial population P_0, we have the equation $P = P_0 e^{kt}$, and k is the population growth rate.

So, for our situation, we would be using the equation $P = 6.056e^{0.017t}$, where t represents the time, in years, beyond the year 2000, and P is measured in millions. Does this work?

Substituting t = 10 (the year 2010): $P = 6.056e^{0.17} \approx 7.178$
Substituting t = 20 (the year 2020): $P = 6.056e^{0.34} \approx 8.508$
Substituting t = 30 (the year 2030): $P = 6.056e^{0.51} \approx 10.085$

It is clear from these calculations that the UN is predicting a growth rate *less than* 1.7% per year.

b. Calculating the growth rate requires us to use the same above formula, but with an unknown for the value of k in each case.

From 2000 to 2010: $6.843 = 6.056e^{10r}$, so $\frac{6.843}{6.056} = e^{10r}$, giving us $\frac{\ln\left(\frac{6.843}{6.056}\right)}{10} = r$, or $r \approx 1.22\%$

From 2010 to 2020: $7.578 = 6.843e^{10r}$, so $\frac{7.578}{6.843} = e^{10r}$, giving us $\frac{\ln\left(\frac{7.578}{6.843}\right)}{10} = r$, or $r \approx 1.02\%$

From 2020 to 2030: $8.199 = 7.578e^{10r}$, so $\frac{8.199}{7.578} = e^{10r}$, giving us $\frac{\ln\left(\frac{8.199}{7.578}\right)}{10} = r$, or $r \approx 0.79\%$