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Effective Mathematics Teaching: One Perspective

Timothy D. Kanold

MUCH of the recent research on effective mathematics teaching focuses on instruction that promotes student activity. This instructional style requires teachers to move away from lecturing and move toward monitoring their students' readiness and checking for understanding. The instructional challenge for each of us as classroom teachers is clear: Go beyond traditional teaching methods. Teaching that articulates goals, promotes strategies for solving problems, and provides student-guided practice is *more important* than the modeling of content skills.

Research on information processing notes the necessity for providing classroom activities that allow students to transfer information from short-term memory to long-term memory as they automate skills (Tobias 1982). Teachers can help with this process by reviewing, rehearsing, and elaborating on the material. Providing time for student reflection and summary in class will enhance student success. The key to the teacher's transition from using "student passivity" to "student activity" instructional methods lies in daily planning. Planning must include preparing for classroom activities that extend beyond lectures only. Seven steps for effective daily planning are given in figure 9.1.

These focal points of daily planning are postulated on two critical beliefs supported in part by research: (1) Students need to practice and process learning *actively*. They need to be actively involved in the review process, the statement of goals, the small-step instruction, and the guided practice. (2) A most critical factor for having an *immediate impact* on student achievement and performance is *monitoring* (Brandt 1984). Walking around during the classroom period is *the* crucial element. Teachers who stay glued to their desks, blackboards, or overheads promote student passivity and management problems. They also limit their ability to diagnose effectively their students' readiness and understanding. Circulating among students helps to reduce disciplinary problems, since the efforts to check for understanding

Planning for Effective Instruction

1. Begin each class with a short overview, review, or "Problem of the Day," as appropriate.
2. Begin each lesson with a short statement of goals and rationale, verbally and in writing.
3. Present new material in small stages, with time for student practice and exploration after each stage.
4. Give clear and detailed instructions.
5. Check for student understanding by posing numerous questions and giving students a lot of active, guided practice.
6. Give specific instructions for seatwork.
7. Allow continued practice in small groups until students are independent and confident.

Fig. 9.1

also monitor the behavior of all students. Thus classroom management becomes secondary. Frequent monitoring during class of students' progress serves as an important diagnostic function, provides readiness checks for the teacher, and allows the teacher to give direct and immediate feedback to students—all of which are known to affect students' achievement and attitudes (Hunter and Russel 1981).

PLANNING THE START OF CLASS

The classroom period should begin with placing on the overhead or the chalkboard one or more review questions or a problem foreshadowing new material. As the students work through this initial activity, the teacher can take attendance, check for passes, monitor students' homework, and answer individual questions in a brisk and timely manner. It is important during this time that the teacher walk quickly around the room checking all students on a regular basis rather than be drawn in by one student for a lengthy period of time. It is expected that all students will begin work on the boardwork activity on entering class, thus bringing them immediately on task. Students are expected to open their notebooks, copy down the exercise(s) or problem of the day, and begin. As the students have completed this work, the teacher then begins class with a short statement of goals, both verbally and in writing, to create an orientation to the lesson and to create the mind-set and rationale necessary for the students to be actively involved in the mathematics. As the goals are stated, list intended outcomes of the lesson. Train students to record these outcomes in their notes each day. It is important to state the outcome as an activity. To ask the students to "understand" the Pythagorean theorem is too vague. To ask them to be able to *state* the theorem, *list* two applications of the theorem, *contrast* its use with 30-60-90 triangles, or *explain* the derivation of the theorem suggests

an activity. Finally, do *not* start class by "going over" homework. Reserve this activity for later in the period.

PRESENTING THE CONTENT AND CHECKING FOR UNDERSTANDING

The move into the presentation of content invites the question, "How do I check for student understanding during the classroom hour?" Most teachers are very good at modeling examples but tend to check for understanding by relying on such verbal cues as, "Did I go too fast for you?" "Isn't this an easy one?" "Okay?" "Everyone see that?" "Who doesn't understand that?" Often included is the rhetorical "Any questions?" These particular cues set up two counterproductive conditions in the classroom because (1) the teacher makes the false assumption that no response indicates that it is okay to continue, and (2) the students develop a sense of lowered self-esteem if they do respond to them, since it is an admission in front of their peers that something is not okay (Johnson 1982). Thus, teachers need to develop successful techniques and methods for checking on students' understanding throughout the classroom period. (These checks also give students an opportunity for momentary reflections on the content being learned.) There are three effective ways to check for understanding:

1. *Reflective Summaries during the Presentation of Content*

On a complex word problem, the teacher could help the students set up the initial investigation of the problem but then allow them time to discuss strategies for solving it. This is a good time to circulate among students to find out if they understand the set-up of the problem and possess the skills necessary to solve it. While walking around, the teacher is receiving and giving feedback and checking for understanding to determine the pace of the lesson. Students can also summarize major points during the class period. After completing several examples for solving systems of two equations in two variables using the elimination method, students could be asked to summarize the major steps for the solution algorithm either in writing or by discussing the process with a partner. Thus, rather than be required to memorize an algorithm that is based on isolated examples, students are given time to understand the underlying steps of the algorithm.

2. *Effective Questioning*

Effective teachers use strategies that encourage *all* students to consider the questions they are asking. Questioning opportunities exist during review, the presentation of content, guided practice, homework activities, and generally during any part of the class period in which the teacher is trying to

assess group awareness, readiness, and understanding. An effective questioning cycle must allow students to listen actively both to the question as well as *to other students' responses*. A questioning cycle likely to result in this active engagement includes the following four steps:

1. Pose the question.
2. Provide "wait time" after each question to prevent student callouts.
3. Select students randomly, making certain to include all students. Call on volunteers as well as nonvolunteers.
4. Redirect the student response to other students for their judgment of correctness or for an extension of an answer.

During questioning periods, some students are reluctant to wait to be called on and like to call out an answer or response. Student callouts can be disruptive because these students then begin to control the pace and the direction of the classroom focus. This particular cycle of questioning reduces student callouts. The use of wait time allows students to think of a response before being called on (Barell 1985). A technique that also encourages wait time is to immediately follow a question with a phrase such as, "Raise your hand when you're reasonably sure of the answer." *Reasonably sure* indicates that it is okay to take a risk and possibly be wrong. The questioning sequence provided below promotes the notion of an effective question cycle.

T: Class, what would be an example of a triangle with area 12 cm^2? Please draw and label a diagram on your papers and raise your hand to respond. [Teacher monitors students as hands are raised.]
T: Only Jessica and Adam have such a triangle? Are there more? Three hands, four hands, anyone else? [Total wait time is eight to ten seconds.]
T: Okay, Roy, I noticed you chose a right triangle. Please explain your diagram to the class. [Roy did not have his hand up.]
R: I drew a right triangle with legs of length 8 cm and 3 cm.
T: How many agree with Roy's example? Raise your hands! Who disagrees with Roy's example? [This forces other students to pay attention to Roy's response.]
T: Who can prove or disprove Roy's assertion . . . Connie? [Connie had her hand up.]
C: In a right triangle, the legs represent a base and height. Thus $A = 1/2bh$ or $A = 1/2(8)(3)$, which is 12 cm^2?
T: Very good! Thank you! As I walked around I noticed all of you used a right triangle. Can you think of an example that is *not* a right triangle? [Dialogue continues.]

This questioning cycle allows teachers to know which students are "with them" during the classroom period. It also increases wait time, limits stu-

dents' callouts, and promotes group feedback to a student response and positive group success on questions asked in class. This style of questioning and group checking for understanding should be used constantly during various classroom activities.

3. *Controlled Guided Practice and Monitoring*

Teachers must plan and provide multiple guided-practice opportunities for students throughout the classroom period. A guided-practice opportunity should follow the teaching of a concept or a procedure. The teacher asks all the students to attempt a particular task while the teacher monitors their work. This time can also be used to test problem-solving strategies. A teacher can give students opportunities to use higher-order thinking skills by asking them to list two or three strategies they might use in solving a problem. As the students begin to make their lists, the teacher observes the ideas they are writing down. In addition to summarizing key ideas or major points in their papers, students can also work in small groups. Guided practice is an excellent time for students to be actively involved in mathematics with each other. Students can work in pairs, with one person explaining a strategy or solution to the other. These math-communication activities can be oriented toward problem solving in activities that require students to "formulate a response," "investigate patterns in the diagram," "develop and apply a strategy for attack," or "verify and interpret results."

PLANNING FOR THE END OF CLASS

Generally one of two conditions exists at the end of a class period: either little time is left and the teacher is scrambling to give the assignment, or ten to fifteen minutes of class time remain. Many teachers feel their "job" is done when the material has been presented. However, it is important for teachers not only to give the assignment but to discuss how to do the assignment. Teachers should review instructions, make sure the directions are clear and precise, and hold the students accountable for their work (McGreal and Collins 1983). The end-of-period time can be an excellent opportunity for teachers to monitor the start of the homework and diagnose any potential problems. It is also a time *to plan* for working with those students who generally do not risk getting involved.

If time is limited, use an end-of-the-hour wrap-up activity. For example, have the students write down in their notes the three major ideas they have learned for the day. They can also talk about the strategies they have used to try to reach the major objectives of the day. *What* is to be learned and *why* they should learn it should not be kept a secret! These wrap-up activities will help students automate content as they develop their own process for learning. Also at this time homework from the previous day can be discussed, graded, corrected, and examined. Key problems can be discussed and reviewed using a group format. Accountability for student homework can be accomplished through (1) collecting notebooks, (2) brief homework quizzes, and (3) collecting isolated problems.

SUMMARY

This article suggests that each teacher use a daily lesson plan that incorporates the following processes:

1. Provide an initial activity that serves either as a review of previous lessons or as an advance organizer for the topic of the present lesson.
2. Explain to students *what* is to be learned and give a rationale for *why* it is to be learned.
3. Allow students to practice new skills and to reflect on the strategies used to perform those skills during the class period. Monitor them and provide feedback as needed.
4. Assign on a daily basis homework that promotes high success rates on using skills and rewards creative thinking on problem-solving activities.
5. Use a questioning cycle that forces students to wait before responding to a question and at the same time uses qualifying statements that allow all students to get involved actively in the question.
6. Interact with the whole class during class time and move students through discussions at a brisk pace with a high level of enthusiasm.
7. Force students to interact and communicate during part of the period.

These ideas do work. They are practical and will have a positive effect on student achievement if combined with a teaching personality imbued with an endless sense of humor and a love of mathematics!

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