Key

MA 232 Lecture Quiz 1 **Version 1**

1. Verify that the function $y=\sqrt{7-t^4}$ is a solution to the differential equation $\frac{dy}{dt}=\frac{-2t^3}{y}$. [5 points]

$y=(7-t^4)^{1/2}$ +1

$\frac{dy}{dt}=\frac{1}{2}(7-t^4)^{-1/2}(-4t^3)$ +2

$=\frac{-2t^3}{\sqrt{7-t^4}}=\frac{-2t^3}{y}$ ✓ +2

→ It's not a problem if they don't show the $\frac{1}{2}$ power as a separate step to start, as long as the process shows they understood it.

2. Write another (different) solution to the differential equation $\frac{dy}{dt}=\frac{-2t^3}{y}$. [2 points]

Anything in the form $y=\sqrt{C-t^4}$ +2

where C is any constant

3. Verify that the function $y=\sin^4 t$ is a solution to the initial value problem $\frac{dy}{dt}=4y\cot t$, with $y\left(\frac{\pi}{4}\right)=\frac{1}{4}$. [8 points]

Verify DE: (5 pts)

$y=(\sin t)^4$

$\frac{dy}{dt}=4(\sin t)^3(\cos t)$ +2

$=4(\sin t)^4\left(\frac{\cos t}{\sin t}\right)$ +1

$=4y\cot t$ ✓ +2

(Note that they could also break down $\frac{dy}{dt}=4y\cot t$ to show equivalence)

Verify I.C. (3 pts)

$y\left(\frac{\pi}{4}\right)\stackrel{?}{=}\frac{1}{4}$

$\left(\sin\frac{\pi}{4}\right)^4\stackrel{?}{=}\frac{1}{4}$ +1 substitute

$(\sin 45)^4\stackrel{?}{=}\frac{1}{4}$ +1 correct angle

$\left(\frac{\sqrt{2}}{2}\right)^4\stackrel{?}{=}\frac{1}{4}$ +1 evaluate

$\frac{1}{4}=\frac{1}{4}$ ✓

(If they don't use degrees, just count eval as +2.)

MA 232 Lecture Quiz 1 **Version 2**

1. Verify that the function $y = \sqrt{9 - t^6}$ is a solution to the differential equation $\frac{dy}{dt} = \frac{-3t^5}{y}$. [5 points]

$y = (9-t^6)^{1/2}$ +1

$\frac{dy}{dt} = \frac{1}{2}(9-t^6)^{-1/2}(-6t^5)$ +2

$= \frac{-3t^5}{\sqrt{9-t^6}} = \frac{-3t^5}{y}$ +2

→ It's not a problem if they don't show the $\frac{1}{2}$ power as a separate step to start, as long as the process shows they understand it.

2. Write another (different) solution to the differential equation $\frac{dy}{dt} = \frac{-3t^5}{y}$. [2 points]

Anything in the form $y = \sqrt{C - t^6}$ +2

where C is any constant.

3. Verify that the function $y = \cos^4 t$ is a solution to the initial value problem $\frac{dy}{dt} = -4y\tan t$, with $y\left(\frac{\pi}{4}\right) = \frac{1}{4}$. [8 points]

Verify DE: (5 pts)

$y = (\cos t)^4$

$\frac{dy}{dt} = 4(\cos t)^3(-\sin t)$ +2

$= 4(\cos t)^4\left(\frac{-\sin t}{\cos t}\right)$ +1

$= -4y\tan t$ ✓ +2

(Note that they could also break down $\frac{dy}{dt} = -4y\tan t$ to show equivalence)

Verify I.C. (3 pts)

$y\left(\frac{\pi}{4}\right) \stackrel{?}{=} \frac{1}{4}$

$\left(\cos\frac{\pi}{4}\right)^4 \stackrel{?}{=} \frac{1}{4}$ +1 substitute

$(\cos 45)^4 \stackrel{?}{=} \frac{1}{4}$ +1 correct angle

$\left(\frac{\sqrt{2}}{2}\right)^4 \stackrel{?}{=} \frac{1}{4}$ +1 evaluate

$\frac{1}{4} = \frac{1}{4}$ ✓ (If they don't use degrees, just count eval as +2)