Lecture Quiz 2 – Solutions – January 25, 2008 – Version 1 @ 1:00 p.m. (on green papers)

1. **4 points for this problem**

$y' = y - t^2$

Find and lightly sketch the isocline for which the slope is 2, marking direction field indicators on it as well.

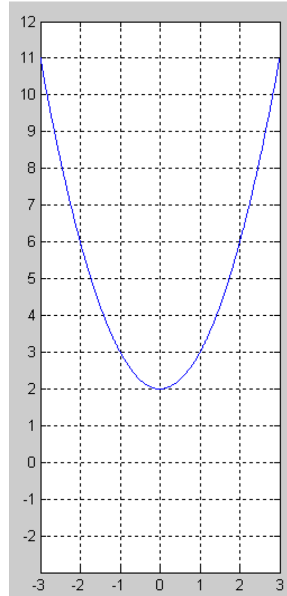
$y' = 2 \Rightarrow y - t^2 = 2 \Rightarrow y = t^2 + 2$ **2 points for this**

The "light sketch" is shown to the right (I'd prefer that it be dotted, but as long as it's drawn properly, that really doesn't matter.)

1 point for this

Along their light sketch, they need to have little segments drawn showing an approximate slope of 2. (As long as it goes uphill to the right, and is clearly steeper than a 1-slope, they're fine.)

1 point for this



2. **7 points for this problem**

Solve by separation of variables: $y' = \dfrac{2t}{1+2y}$ with $y(1) = -2$.

Rewrite and separate: $\dfrac{dy}{dt} = \dfrac{2t}{1+2y} \Rightarrow (1+2y)dy = 2t\,dt$ **2 points for this**

Integrate both sides: $\int (1+2y)dy = \int 2t\,dt \Rightarrow y + y^2 = t^2 + C$ **2 points for this**

Apply initial condition (1, –2): $-2 + (-2)^2 = 1^2 + C \Rightarrow C = 1$ **2 points for this**

Write final (implicit) solution: $y + y^2 = t^2 + 1$ (or equivalent) **1 point for this**

3. **6 points for this problem**

IVP $y' = y - 2t$, where $y(1) = -2$, approximate $y(2)$ using Euler's method, with $h = \dfrac{1}{2}$

Calculate slope at (1, –2): –2 – 2(1) = –4 **1 point for this**

Find first approximation: $t_{k+1} = t_k + \dfrac{1}{2}$ and $y_{k+1} = y_k + h(f(t_k, y_k))$

So, new point is $\left(\dfrac{3}{2}, -4\right)$ **2 points for this**

Calculate slope at $\left(\dfrac{3}{2}, -4\right)$: $-4 - 2\left(\dfrac{3}{2}\right) = -7$ **1 point for this**

Find the desired approximation: new point is $\left(2, -\dfrac{15}{2}\right)$ **2 points for this**

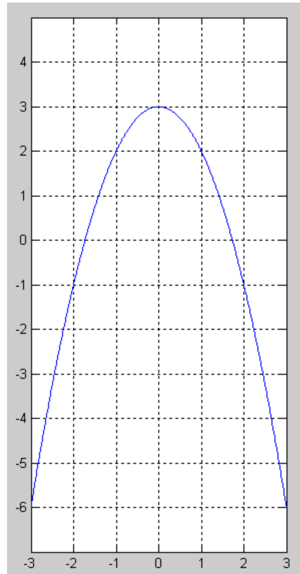
(*Technically, the answer is just the –7.5, but if they don't identify it separately, that's okay.)*

Lecture Quiz 2 – Solutions – January 25, 2008 – Version 2 @ 3:00 p.m. (on ivory papers)

1. **4 points for this problem**

$y' = y + t^2$

Find and lightly sketch the isocline for which the slope is 3, marking direction field indicators on it as well.



$y' = 3 \Rightarrow y + t^2 = 3 \Rightarrow y = 3 - t^2$ **2 points for this**

The "light sketch" is shown to the right (I'd prefer that it be dotted, but as long as it's drawn properly, that really doesn't matter.)

1 point for this

Along their light sketch, they need to have little segments drawn showing an approximate slope of 3. (As long as it goes uphill to the right, and is clearly steeper than a 1-slope, they're fine.)

1 point for this

2. **7 points for this problem**

Solve by separation of variables: $y' = \dfrac{3t^2}{1-2y}$ with $y(1) = -2$.

Rewrite and separate: $\dfrac{dy}{dt} = \dfrac{3t^2}{1-2y} \Rightarrow (1-2y)dy = 3t^2\,dt$ **2 points for this**

Integrate both sides: $\int (1-2y)dy = \int 3t^2\,dt \Rightarrow y - y^2 = t^3 + C$ **2 points for this**

Apply initial condition (1, –2): $-2-(-2)^2 = 1^3 + C \Rightarrow C = -7$ **2 points for this**

Write final (implicit) solution: $y + y^2 = t^2 - 7$ (or equivalent) **1 point for this**

3. **6 points for this problem**

IVP $y' = y + 4t$, where $y(1) = -2$, approximate $y(2)$ using Euler's method, with $h = \dfrac{1}{2}$

Calculate slope at (1, –2): –2 + 4(1) = 2 **1 point for this**

Find first approximation: $t_{k+1} = t_k + \dfrac{1}{2}$ and $y_{k+1} = y_k + h\big(f(t_k, y_k)\big)$

So, new point is $\left(\dfrac{3}{2}, -1\right)$ **2 points for this**

Calculate slope at $\left(\dfrac{3}{2}, -1\right)$: $-1 + 4\left(\dfrac{3}{2}\right) = 5$ **1 point for this**

Find the desired approximation: new point is $\left(2, \dfrac{3}{2}\right)$ **2 points for this**

(*Technically, the answer is just the 1.5, but if they don't identify it separately, that's okay.)*