Lecture Quiz 3 Solutions – Monday, February 4, 2008 **Blue version**

1. Solve the DE: $\frac{dy}{dt}+\frac{4}{t}y=\frac{1}{t^4}$. [8 points total]

Common sense will dictate that integrating factor should be used…

Find the integrating factor: $\mu(t)=e^{\int p(t)\,dt}=e^{\int \frac{4}{t}dt}=e^{4\ln|t|}=t^4$ **2 points for this**
(*they <u>must</u> show an absolute value*)

Use the integrating factor: $t^4\left(y'+\frac{4}{t}y\right)=1$ **1 point for this**

Modify the left side: $\frac{d}{dt}\left(t^4y\right)=1\Rightarrow d\left(t^4y\right)=dt$ **1 point for this**

Integrate both sides: $\int d\left(t^4y\right)=\int dt\Rightarrow t^4y=t+C$ **2 points for this**
(*they <u>must</u> have a constant*)

Solve for *y*: $y=\frac{1}{t^3}+\frac{C}{t^4}$ or $y=\frac{t+C}{t^4}$ **2 points for this**

2. Is $y'''-3ty'+2y^2=t^3$ linear or nonlinear? What is the order of the DE? [2 points total]

This equation is nonlinear. **1 point**
Its order is three (or third). **1 point**

3. A radioactive substance has a half-life of 3 days. We begin with 12 g of this substance. How much of the substance remains after 1 week (*nearest tenth*)? [8 points total]

The standard growth equation is $y=y_0e^{kt}$ **2 points**
(in case full development is shown)

We start with 12 g, so $y=12e^{kt}$ **1 point**

Use half-life to find *k*: $6=12e^{3k}\Rightarrow e^{3k}=\frac{1}{2}\Rightarrow k=\frac{-\ln 2}{3}$,
so $k\approx$ –0.23104906018664843647241070715273 **3 points**

Equation becomes $y=12e^{-0.231t}$ …
Find *y* when *t* = 7: $y=12e^{-0.231(7)}$
$\approx 2.3811015779522992121275584589085$ or 2.4 g **2 points**

As specified in the syllabus, two points are deducted for missing name, ID, or recitation section number. Thus, these last two points for required information make this a 20 point quiz.

Lecture Quiz 3 Solutions – Monday, February 4, 2008 **Green version**

1. Solve the DE: $\frac{dy}{dt}+\frac{2}{t}y=\frac{1}{t^2}$. [8 points total]

Common sense will dictate that integrating factor should be used…

Find the integrating factor: $\mu(t)=e^{\int p(t)\,dt}=e^{\int\frac{2}{t}dt}=e^{2\ln|t|}=t^2$ **2 points for this**
(*they must show an absolute value*)

Use the integrating factor: $t^2\left(y'+\frac{2}{t}y\right)=1$ **1 point for this**

Modify the left side: $\frac{d}{dt}\left(t^2y\right)=1\Rightarrow d\left(t^2y\right)=dt$ **1 point for this**

Integrate both sides: $\int d\left(t^2y\right)=\int dt\Rightarrow t^2y=t+C$ **2 points for this**
(*they must have a constant*)

Solve for *y*: $y=\frac{1}{t}+\frac{C}{t^2}$ or $y=\frac{t+C}{t^2}$ **2 point for this**

2. Is $y''+4ty'-3y=t^3$ linear or nonlinear? What is the order of the DE? [2 points total]

This equation is linear. **1 point**
Its order is two (or second). **1 point**

3. A radioactive substance has a half-life of 5 days. We begin with 16 g of this substance. How much of the substance remains after 2 weeks (*nearest tenth*)? [8 points total]

The standard growth equation is $y=y_0e^{kt}$ **2 points**
(in case full development is shown)

We start with 16 g, so $y=16e^{kt}$ **1 point**

Use half-life to find *k*: $8=16e^{5k}\Rightarrow e^{5k}=\frac{1}{2}\Rightarrow k=\frac{-\ln 2}{5}$,
so $k\approx$ –0.13862943611198906188344642429164 **3 points**

Equation becomes $y=16e^{-0.1386t}$ …
Find *y* when *t* = 14: $y=16e^{-0.1386(14)}$
$\approx 2.2973967099940700135972538935559$ or 2.3 g **2 points**

As specified in the syllabus, two points are deducted for missing name, ID, or recitation section number. Thus, these last two points for required information make this a 20 point quiz.

Lecture Quiz 3 Solutions – Monday, February 4, 2008 **Brown version**

1. Solve the DE: $\frac{dy}{dt}+\frac{3}{t}y=\frac{1}{t^3}$. [8 points total]

Common sense will dictate that integrating factor should be used…

Find the integrating factor: $\mu(t)=e^{\int p(t)\,dt}=e^{\int\frac{3}{t}dt}=e^{3\ln|t|}=t^3$ **2 points for this**
(*they <u>must</u> show an absolute value*)

Use the integrating factor: $t^3\left(y'+\frac{3}{t}y\right)=1$ **1 point for this**

Modify the left side: $\frac{d}{dt}\left(t^3y\right)=1\Rightarrow d\left(t^3y\right)=dt$ **1 point for this**

Integrate both sides: $\int d\left(t^3y\right)=\int dt\Rightarrow t^3y=t+C$ **2 points for this**
(*they <u>must</u> have a constant*)

Solve for *y*: $y=\frac{1}{t^2}+\frac{C}{t^3}$ or $y=\frac{t+C}{t^3}$ **2 point for this**

2. Is $y''-3ty'+2y=t^3$ linear or nonlinear? What is the order of the DE? [2 points total]

This equation is linear. **1 point**
Its order is two (or second). **1 point**

3. A radioactive substance has a half-life of 2 days. We begin with 12 g of this substance. How much of the substance remains after 1 week (*nearest tenth*)? [8 points total]

The standard growth equation is $y=y_0e^{kt}$ **2 points**
(in case full development is shown)

We start with 12 g, so $y=12e^{kt}$ **1 point**

Use half-life to find *k*: $6=12e^{2k}\Rightarrow e^{2k}=\frac{1}{2}\Rightarrow k=\frac{-\ln 2}{2}$,
so $k\approx -0.346573590279972654708616060729 09$ **3 points**

Equation becomes $y=12e^{-0.3466t}$…
Find *y* when *t* = 7: $y=12e^{-0.3466(7)}$
$\approx 1.060660171779821286601266543157 3$ or 1.1 g **2 points**

As specified in the syllabus, two points are deducted for missing name, ID, or recitation section number. Thus, these last two points for required information make this a 20 point quiz.

Lecture Quiz 3 Solutions – Monday, February 4, 2008 **Purple version**

1. Solve the DE: $\frac{dy}{dt}+\frac{5}{t}y=\frac{1}{t^5}$. [8 points total]

Common sense will dictate that integrating factor should be used…

Find the integrating factor: $\mu(t)=e^{\int p(t)\,dt}=e^{\int\frac{5}{t}dt}=e^{5\ln|t|}=t^5$ **2 points for this**
(*they <u>must</u> show an absolute value*)

Use the integrating factor: $t^5\left(y'+\frac{5}{t}y\right)=1$ **1 point for this**

Modify the left side: $\frac{d}{dt}\left(t^5y\right)=1\Rightarrow d\left(t^5y\right)=dt$ **1 point for this**

Integrate both sides: $\int d\left(t^5y\right)=\int dt\Rightarrow t^5y=t+C$ **2 points for this**
(*they <u>must</u> have a constant*)

Solve for *y*: $y=\frac{1}{t^4}+\frac{C}{t^5}$ or $y=\frac{t+C}{t^5}$ **2 point for this**

2. Is $y'''+4ty'-3y^2=t^3$ linear or nonlinear? What is the order of the DE? [2 points total]

This equation is nonlinear. **1 point**
Its order is two (or second). **1 point**

3. A radioactive substance has a half-life of 4 days. We begin with 16 g of this substance. How much of the substance remains after 2 weeks (*nearest tenth*)? [8 points total]

The standard growth equation is $y=y_0e^{kt}$ **2 points**
(in case full development is shown)

We start with 16 g, so $y=16e^{kt}$ **1 point**

Use half-life to find *k*: $8=16e^{4k}\Rightarrow e^{4k}=\frac{1}{2}\Rightarrow k=\frac{-\ln 2}{4}$,

So $k\approx -0.17328679513998632735430803036454$ **3 points**

Equation becomes $y=16e^{-0.1733t}$…

Find *y* when *t* = 14: $y=16e^{-0.1733(14)}$

$\approx 1.4142135623730950488016887242097$ or 1.4 g **2 points**

As specified in the syllabus, two points are deducted for missing name, ID, or recitation section number. Thus, these last two points for required information make this a 20 point quiz.