About half for setup, half for solution

Version 1 – green paper, "Left"
A 300-liter tank contains 100 liters of fresh water. A brine solution containing 2 g of salt per liter enters the tank at 3 liters per minute, while the well-mixed solution is drained off at 1 liter per minute. How many grams of salt are in the tank at the moment when the tank fills completely?

Let x = grams of salt in the tank after t minutes *(I'll actually give them this)* **+1 point**

$\frac{dx}{dt}$ = rate in − rate out (also provided) +1

rate in = (conc in)(flow in) = $(2\ g/l)(3\ l/min) = 6\ g/min$ +2

rate out = (conc out)(flow out) = $\left(\frac{x}{100+2t}\ g/l\right)(1\ l/min) = \frac{x}{100+2t}\ g/min$ +3

So, $\frac{dx}{dt} = 6 - \frac{x}{100+2t}$ +1

Linear structure, so $x' + \left(\frac{1}{100+2t}\right)x = 6$, with $x(0)=0$ +2

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200 gal to fill, net of +2 gal/min, so 100 min to fill +2

Integrating factor $\mu(t) = e^{\int \frac{1}{100+2t}dt} = e^{\frac{1}{2}\ln|100+2t|} = \sqrt{100+2t}$ +2

$\sqrt{100+2t}\left(x' + \frac{1}{100+2t}x\right) = 6\sqrt{100+2t}$ +1

$\int d\left(x\sqrt{100+2t}\right) = \int 6\sqrt{100+2t}\ dt$ +1

$x\sqrt{100+2t} = 2(100+2t)^{3/2} + C$

$x = 2(100+2t) + \frac{C}{\sqrt{100+2t}}$ +1

$x(0)=0$, so $C = -2000$

$x(t) = 2(100+2t) - \frac{2000}{\sqrt{100+2t}}$ +1

Find $x(100\text{ min}) = 2(300) - \frac{2000}{\sqrt{300}} \approx 484.5299$ grams +2

See 1st sheet for points details

Version 2 – green paper, "Right"
A 300-liter tank contains 150 liters of fresh water. A brine solution containing 2 g of salt per liter enters the tank at 3 liters per minute, while the well-mixed solution is drained off at 1 liter per minute. How many grams of salt are in the tank at the moment when the tank fills completely?

Let x = grams of salt in the tank after t minutes *(I'll actually give them this)* **+1 point**

$\frac{dx}{dt}$ = rate in − rate out

$(\text{rate in}) = \left(2\,\text{g}/\ell\right)\left(3\,\ell/\text{min}\right) = 6\ \text{g/min}$

$(\text{rate out}) = \left(\frac{x\ \text{g}}{(150+2t)\ \ell}\right)\left(1\ \ell/\text{min}\right) = \frac{x}{150+2t}\ \text{g/min}$

So, $\frac{dx}{dt} = 6 - \frac{x}{150+2t}$

$x' + \left(\frac{1}{150+2t}\right)x = 6, \quad x(0) = 0$

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150 gal to fill, net of +2 gal/min, so 75 min to fill

Int. fact: $\mu(t) = e^{\int \frac{1}{150+2t}dt} = \sqrt{150+2t}$

$\sqrt{150+2t}\left(x' + \frac{1}{150+2t}x\right) = 6\sqrt{150+2t}$

$\int d\left(x\sqrt{150+2t}\right) = \int 6\sqrt{150+2t}\,dt$

$x\sqrt{150+2t} = 2(150+2t)^{3/2} + C$

$x = 2(150+2t) + \frac{C}{\sqrt{150+2t}}$

$x(0) = 0$, so $C = -300\sqrt{150}$

$x(t) = 2(150+2t) - \frac{300\sqrt{150}}{\sqrt{150+2t}}$

Find $x(75) = 2(300) - \frac{300\sqrt{150}}{\sqrt{300}}$

$\simeq 387.868$ g

See 1st sheet for points details

Version 3 – ivory paper, "Left"
A 400-liter tank contains 100 liters of fresh water. A brine solution containing 2 g of salt per liter enters the tank at 3 liters per minute, while the well-mixed solution is drained off at 1 liter per minute. How many grams of salt are in the tank at the moment when the tank fills completely?

Let x = grams of salt in the tank after t minutes *(I'll actually give them this)* **+1 point**

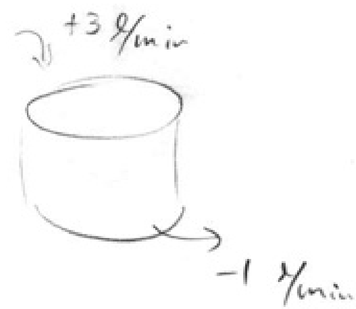
$\frac{dx}{dt}$ = rate in − rate out

rate in $= \left(2\,\text{g}/\ell\right)\left(3\,\ell/\text{min}\right) = 6\,\text{g}/\text{min}$

rate out $= \left(\frac{x\ \text{g}}{100+2t\ \ell}\right)\left(\frac{1\ \ell}{\text{min}}\right) = \frac{x}{100+2t}\ \text{g}/\text{min}$

So, $\frac{dx}{dt} = 6 - \frac{x}{100+2t}$

$x' + \left(\frac{1}{100+2t}\right)x = 6, \quad x(0) = 0$



+2

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300 gal to fill, net +2 gal/min, so 150 min to fill

Int fact: $\mu(t) = \sqrt{100+2t}$ (same as V1)

$x(t) = 2(100+2t) - \frac{2000}{\sqrt{100+2t}}$

Find $x(150) = 2(400) - \frac{2000}{\sqrt{400}}$

$= 800 - 100$

$= 700$ grams

See 1st sheet for points details

Version 4 – ivory paper, "Right"
A 400-liter tank contains 150 liters of fresh water. A brine solution containing 2 g of salt per liter enters the tank at 3 liters per minute, while the well-mixed solution is drained off at 1 liter per minute. How many grams of salt are in the tank at the moment when the tank fills completely?

Let x = grams of salt in the tank after t minutes *(I'll actually give them this)* **+1 point**

$$\frac{dx}{dt} = \text{rate in} - \text{rate out}$$

$$\text{rate in} = \left(2\,g/\ell\right)\left(3\,\ell/\text{min}\right) = 6\ g/\text{min}$$

$$\text{rate out} = \left(\frac{x\ g}{(150+2t)\ \ell}\right)\left(\frac{1\ \ell}{\text{min}}\right) = \frac{x}{150+2t}\ g/\text{min}$$

$$\text{So } \frac{dx}{dt} = 6 - \frac{x}{150+2t}$$

$$x' + \left(\frac{1}{150+2t}\right)x = 6, \quad x(0)=0$$

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250 gal to fill, net of +2 gal/min, so 125 min to fill

$$\text{Int fact}: \mu(t) = e^{\int \frac{1}{150+2t}dt} = \sqrt{150+2t} \quad (\text{same as V2})$$

$$x(t) = 2(150+2t) - \frac{300\sqrt{150}}{\sqrt{150+2t}}$$

$$\text{Find } x(125) = 2(400) - \frac{300\sqrt{150}}{\sqrt{400}}$$

$$= 800 - 15\sqrt{150}$$

$$\approx 616.288\ g$$