MA-131 Recitation Review for Final Exam

① $f(x) = (4x+1)^2$

Definition of Derivative: $f'(x) = \lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$$f'(x) = \lim_{h \to 0} \frac{(4(x+h)+1)^2 - (4x+1)^2}{h} = \lim_{h \to 0} \frac{16(x+h)^2 + 8(x+h) + 1 - 16x^2 - 8x - 1}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{16x^2} + 32xh + 16h^2 + \cancel{8x} + 8h + \cancel{1} - \cancel{16x^2} - \cancel{8x} - \cancel{1}}{h} = \lim_{h \to 0} \frac{32x\cancel{h} + \overset{\to 0}{16h^2} + 8\cancel{h}}{\cancel{h}} = 32x + 8$$

So, $f'(-1) = 32(-1) + 8 = -24$

② $f(x) = \ln(4x^2)$, $f'(x) = \dfrac{1}{4x^2}(8x) = \dfrac{2}{x}$

③ $y = \sin^{-1}(x) \Longrightarrow \dfrac{dy}{dx} = \dfrac{1}{\sqrt{1-x^2}}$

④ $f(t) = \cot(2t) \Longrightarrow f'(t) = \sec^2(2t) \cdot 2 = 2\sec^2(2t)$

⑤ $f(x) = \dfrac{x-1}{x^2+2}$ $\quad \dfrac{gf' - fg'}{g^2}$

$$f'(x) = \frac{(x^2+2)(1) - (x-1)(2x)}{(x^2+2)^2} = \frac{x^2+2 - 2x^2 + 2x}{(x^2+2)^2} = \frac{-x^2+2x+2}{(x^2+2)^2}$$

(6) $g(x) = \sec^4(\cos(x^2)) = \overbrace{(\underbrace{\sec(\overbrace{\cos(\underbrace{x^2}_{4})}^{3})}_{2})^4}^{1}$

$$g'(x) = \underbrace{4(\sec(\cos(x^2)))^3}_{1}\underbrace{(\sec(\cos(x^2))\tan(\cos(x^2)))}_{2}\underbrace{(-\sin(x^2))}_{3}\underbrace{(2x)}_{4}$$

$$= -8x(\sec(\cos(x^2)))^4(\tan(\cos(x^2)))(\sin(x^2))$$

(7) $f(x) = \dfrac{x^2-16}{2x^2+7x-4}$

A) Discontinuity when $2x^2+7x-4=0$. Factor to: $(2x-1)(x+4)$

$\underbrace{x = \frac{1}{2}}_{\text{Vertical Asymptote}}, x = -4$

B) $x^2-16 = (x+4)(x-4) = 0$

$x=4; x=-4$ $\therefore$ Removable Discontinuity @ $\underline{x=-4}$

So, $f(x) = \dfrac{\cancel{(x+4)}(x-4)}{(2x-1)\cancel{(x+4)}} \Rightarrow f(-4) = \dfrac{-8}{-9} = \dfrac{8}{9}$

Define $f(-4) = 8/9$ for continuity @ -4. $\square$

(8) $\lim\limits_{x\to\infty} \dfrac{1}{\cos(\underbrace{1/x}_{\to 0 \text{ as } x\to\infty})} = \dfrac{1}{1} = \boxed{1}$

$\cos(0)=1$

(9) $\lim\limits_{x\to\infty} \dfrac{\sqrt{x^4-2}}{x^2+3}$

Divide by highest power in denominator. $\frac{1}{x^2} = \sqrt{\frac{1}{x^4}}$

$$\lim_{x\to\infty} \frac{\frac{1}{x^2}\sqrt{x^4-2}}{1+\underbrace{\cancel{\frac{3}{x^2}}}_{0}} = \lim_{x\to\infty} \frac{\sqrt{1-\cancel{\frac{2}{x^4}}^{\to 0}}}{1} = \frac{1}{1} = \boxed{1}$$

(10) Correction: $F(x) = \dfrac{3x}{\sin(x)}$ $\left[\underline{\underline{\text{NOT}}}\ F(x) = \dfrac{3x+1}{\sin(x)}\right]$

$$\lim_{x\to 0} \frac{3x}{\sin(x)} = 3\lim_{x\to 0} \frac{x}{\sin(x)}$$

Solution 1: Since $\lim\limits_{x\to 0} \frac{\sin(x)}{x} = 1$, $3\lim\limits_{x\to 0} \frac{x}{\sin(x)} = 3\lim\limits_{x\to 0} \frac{1}{\frac{\sin(x)}{x}} = 3\cdot\frac{1}{1} = \boxed{3}$

Solution 2: <u>L'hopital</u>

$$\lim_{x\to 0} \frac{3x}{\sin(x)} = \lim_{x\to 0} \frac{3}{\underbrace{\cos(x)}_{\substack{1 \\ \text{as } x\to 0}}} = \frac{3}{1} = \boxed{3}$$