Sample Final Problem 4

Inverse LaPlace transform of $G(s) = \dfrac{2s+16}{s^2+4s+13}$

Always check to see if the denominator is factorable, since you could use partial fraction separation. In this case, that is not possible, so I'll perform completing the square in the denominator: $\dfrac{2s+16}{s^2+4s+4+9}$, which I'll then re-write as $\dfrac{2s+16}{(s+2)^2+3^2}$. Now, once I see the structure of the denominator, I can "go to work" on the numerator: $\dfrac{2s+4+12}{(s+2)^2+3^2} = 2\left(\dfrac{s+2}{(s+2)^2+3^2}\right)+\left(\dfrac{12}{(s+2)^2+3^2}\right) = 2\left(\dfrac{s+2}{(s+2)^2+3^2}\right)+4\left(\dfrac{3}{(s+2)^2+3^2}\right)$. I'm now ready to do the inverse transform:

$$\mathscr{L}^{-1}\left\{\frac{2s+16}{s^2+4s+13}\right\} = 2\mathscr{L}^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} + 4\mathscr{L}^{-1}\left\{\frac{3}{(s+2)^2+3^2}\right\} = 2e^{-2t}\cos 3t + 4e^{-2t}\sin 3t\,.$$