Sample Final Problem 6

Finding eigenpairs for $\mathbf{A} = \begin{bmatrix} 4 & 2 \\ -3 & 11 \end{bmatrix}$

<u>Find eigenvalues</u> ~4 points

Eigenvalues are found using $\begin{vmatrix} 4-\lambda & 2 \\ -3 & 11-\lambda \end{vmatrix} = 0 \Rightarrow (4-\lambda)(11-\lambda)-(-6)=0 \Rightarrow \lambda^2 - 15\lambda + 50 = 0$. This quadratic factors nicely, so $(\lambda-10)(\lambda-5)=0$, and clearly $\lambda_1 = 5$ and $\lambda_2 = 10$. (Obviously my choice of subscript order is arbitrary.)

<u>First eigenpair</u> ~3 points

Finding an eigenvector for λ_1: we know that $\begin{bmatrix} -1 & 2 \\ -3 & 6 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, augmented matrix would be $\left[\begin{array}{cc|c} -1 & 2 & 0 \\ -3 & 6 & 0 \end{array}\right]$, and in RREF would be $\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right]$. The relationship given is $v_1 - 2v_2 = 0 \Rightarrow v_1 = 2v_2$, so I pick something simple like $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$. My first eigenpair is $\left(5, \begin{bmatrix} 2 \\ 1 \end{bmatrix}\right)$.

<u>Second eigenpair</u> ~3 points

Finding an eigenvector for λ_2: we know that $\begin{bmatrix} -6 & 2 \\ -3 & 1 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, augmented matrix would be $\left[\begin{array}{cc|c} -6 & 2 & 0 \\ -3 & 1 & 0 \end{array}\right]$, in RREF would be $\left[\begin{array}{cc|c} 1 & -1/3 & 0 \\ 0 & 0 & 0 \end{array}\right]$. Relationship we get is $v_1 - \frac{1}{3}v_2 = 0 \Rightarrow v_1 = \frac{1}{3}v_2 \Rightarrow 3v_1 = v_2$; I choose a simple vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Second eigenpair is $\left(10, \begin{bmatrix} 1 \\ 3 \end{bmatrix}\right)$.