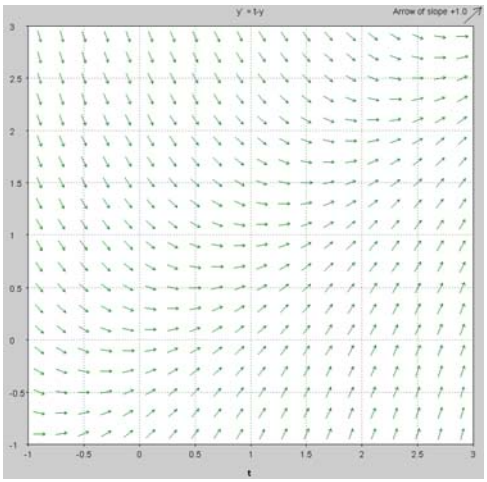
MA 232 – Fall 2007 – Samples of problems on First Test

1. Write (but do not solve) a first-order differential equation to express the following situation: *a college student finds that the rate of increase of his course average A is directly proportional to the number of hours he studies per week (H), and inversely proportional to the number of hours he spends playing Guitar Hero (G).*

2. For this problem, let *s* represent the height of the marble off the ground at time *t*, let *v* represent its velocity at time *t*, and let *a* represent its acceleration at time *t*. Recall that $\frac{ds}{dt} = v$ and $\frac{dv}{dt} = a$. *Suppose a marble is dropped (not thrown) off the top of the Empire State Building's lightning rod, 1,454 feet off the ground. In absence of friction or air resistance, with what velocity will it hit the ground below, assuming no obstacles? Recall also that acceleration due to gravity is* $-32.2 \frac{\text{ft}}{\text{sec}^2}$.

3. The accompanying diagram shows the direction field for the differential equation $y' = t - y$. Perform additional qualitative analysis (including checking for equilibria and investigating some isoclines) to share some conclusions about the nature of the solutions of this equation. Draw sample solution curves on the diagram itself.



4. Solve the following initial value problem using separation of variables: $y' = \frac{2t}{1+2y}$, $y(2) = 0$.

5. Find all equilibria for the autonomous differential equation $y' = 3y - y^2$, and check the stability of each one that you find.

6. Given the differential equation $y' = y^2 - 4t$, find the second derivative y'' and then set it equal to zero to determine potential points of inflection. Write the equation for this answer solved for *t* for convenience.

7. For the equation $y' = \sqrt{t + y}$, with $y(1) = 3$, use Euler's method with step size of 0.5 to find an approximation for $y(2)$. Also, find an approximation for $y(2)$ using Runge-Kutta, second order, with step size of 0.5. Compare the results between the two methods.

8. Classify the following equations as linear or nonlinear. If linear, then specify whether the equation is homogeneous or nonhomogeneous. In any case, state the order of the differential equation.

a. $e^t \frac{d^2y}{dt^2} + 2t\frac{dy}{dt} - t^2 y = 0$ *b.* $y''' + yty' - 2t^2 = 4t$

9. Solve the linear differential equation $y' - y = e^{3t}$ using the integrating factor method. Solve the same equation using the Euler-Lagrange process.

10. For the growth problem $y' = ky$, with $k > 0$, the time required for the amount of substance to double (t_D) is called the doubling time. If $y = y_0$ when $t = 0$, determine t_D in terms of k.

11. In 1820, a William Record in London deposited the equivalent of $0.50 for his granddaughter in the Bank of London. Unfortunately, he died before telling his granddaughter of the account. In 1980, the heirs of the granddaughter discovered the account, which had gained 6% annual interest, compounded continuously. How much money was in the account at that time?

12. A 600-gallon tank is filled with 300 gal of pure water. A spigot is opened and a salt solution containing 1 lb of salt per gallon of solution begins flowing into the tank at a rate of 3 gal/min. Simultaneously, a drain is opened at the bottom of the tank, allowing the well-mixed solution to leave the tank at a rate of 1 gal/min. What will be the salt content in the tank at the precise moment that the volume of solution in the tank reaches the tank's capacity of 600 gal?

13. In a city with a population of 200,000, a rumor about dihydrogen monoxide in the drinking water began to spread one day. After one week, 1,000 had been alarmed by the news. Assume that the rate of increase of the number N of people who have heard the rumor is proportional to the product of those who have heard the rumor and those who have not heard the rumor. Further, suppose that when $t = 0$, $N = 1$ (that is, a single person began the rumor).

    a. Find an expression for $\frac{dN}{dt}$ – write it in a logistic form.
    b. Find the equilibrium solutions, and identify the carrying capacity.
    c. Determine how many days it will take (from the start of the rumor) for half of the population to know about the rumor.

14. Consider the system of differential equations: $\frac{dx}{dt} = y$ and $\frac{dy}{dt} = 5x + 3y$.

    a. Determine and plot the equilibrium points and nullclines for the system.
    b. Using the regions of the plane created by the nullclines, determine general vector field information in each region; recall that $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. Determine whether each equilibrium point is stable or unstable from your findings.

15. **Unfinished problem from class…**
    A tank initially contains 100 gallons of pure water. Water with a dissolved salt concentration of 1 gram per gallon begins to flow in at a rate of 2 gallons per minute. At the same time, the well-mixed salt solution in the tank is being pumped out at a rate of 3 gallons per minute. How many minutes does it take for the amount of salt in the tank to reach a maxiumum?

Make use of these samples (and others that will be posted soon) to create your half-sheet of notes that you may use during the exam. If there are any other items that you think would be helpful, include them as well. Remember that you may use both sides of the half-sheet of paper (5 ½ x 8 ½ inches).