MA 131 Calculus I – Spring 2008

Written Homework 2
Due by Friday, February 29, 2008 at the start of lecture.
Late homework is not accepted.

A particle is moving horizontally along a straight line. The line is marked (like a number line would be) with numerical values appropriately spaced along it.

At any time $t \geq 0$, the position of the particle on the marked line is given by $s(t) = t^4 - 13t^2 + 24$.

a. What is the value of *s*(0)? Explain the physical interpretation of this result. **[3 pts, 5 pts]**

The value of *s*(0) is 24. This is telling us that when we "begin the clock" for this experiment, the particle is at the location on the straight line marked with 24.

Numerical answer: *s*(0) = 24.

b. What is the value of *s*(2)? What is the average velocity of the particle in the first two seconds? Physically, explain what this means, and how you determined it. **[3 pts, 5 pts, 5 pts]**

The value of *s*(2) is 16 – 52 + 24 = –12. This tells us that after two seconds, the particle is at the location on the line marked –12, meaning the net movement of the particle in two seconds is to the left 36 units. From this, I can calculate the average velocity by dividing –36 by 2, which gives me –18 units per second.

More formally, this could be a slope formula calculation (difference quotient), as follows:

$$\bar{v} = \frac{s(2) - s(0)}{2 - 0} = \frac{-12 - 24}{2} = \frac{-36}{2} = -18.$$ Units must then be added.

Numerical answers: *s*(2) = –12 and average velocity is –18 units per second.

c. Find a function *v*(*t*) that can be used, for $t \geq 0$, to evaluate the velocity of the particle at time *t*. Explain how you determined this, and why. **[5 pts, 5 pts]**

I know that the velocity is the instantaneous rate of change of position, so *v*(*t*) = *s*′(*t*). I'll take the derivative of the function they gave me, and get $v(t) = 4t^3 - 26t$.

Numerical answer: $v(t) = 4t^3 - 26t$.

d. Are there any times when the particle is not moving? Explain. **[5 pts, 5 pts]**

Any time when the particle is not moving would mean the velocity function (found above) has to be 0. So, I will set $v(t) = 0$ and see if there are any such times. $4t^3 - 26t = 0$, which when we factor gives us $2t(2t^2 - 13) = 0$, so $t = 0$ is clearly one time (at the start), and also when $2t^2 - 13 = 0$, or $2t^2 = 13 \Rightarrow t^2 = \frac{13}{2} \Rightarrow t = \pm\sqrt{\frac{13}{2}}$. Now, from the original problem, we know that $t \geq 0$, so we can ignore the negative value.

Numerical answer: $t = 0, \sqrt{\frac{13}{2}}$.

e. What is the speed of the particle after 2 seconds? How did you determine this? **[4 pts, 5 pts]**

To find this, I can simply plug 2 in the function for velocity. *If it comes out negative, I must make it positive, since speed can only be positive.* So, $v(2) = 4(2^3) - 26(2) = 32 - 52 = -20$, and so my answer is 20 units per second.

Numerical answer: 20 units per second.